Discovering Roles and Anomalies in Graphs: Theory and Applications

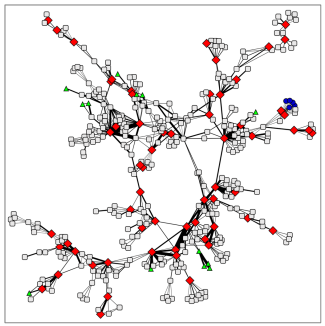
Part 2: Patterns and Anomalies

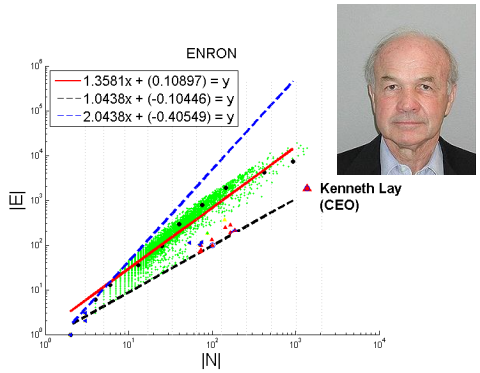
Tina Eliassi-Rad (Rutgers)

Christos Faloutsos (CMU)

ECML PKDD 2013 Tutorial

OVERVIEW - high level:





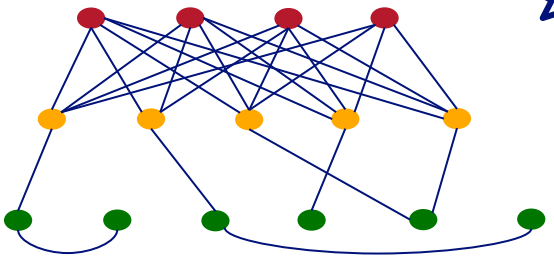
Features

Roles

Anomalies

= rare roles

Patterns



Resource:

Open source system for mining huge graphs:

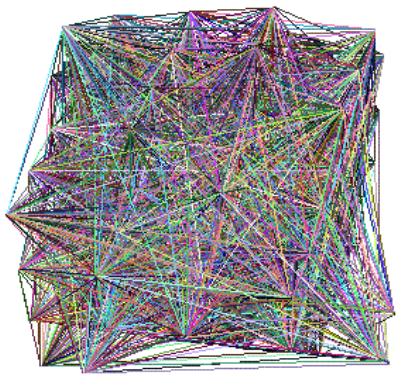
PEGASUS project (PEta GrAph mining System)

- www.cs.cmu.edu/~pegasus
- code and papers

Roadmap

- Patterns in graphs
  - Overview
  - Static graphs
  - Weighted graphs
  - Time-evolving graphs
- Anomaly Detection
- Application: ebay fraud
- Conclusions

Problem #1 - network and graph mining



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- What does FaceBook look like?

- What is 'normal' / 'abnormal' ?
- which patterns/laws hold?

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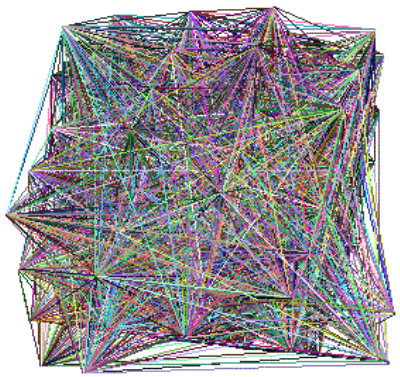

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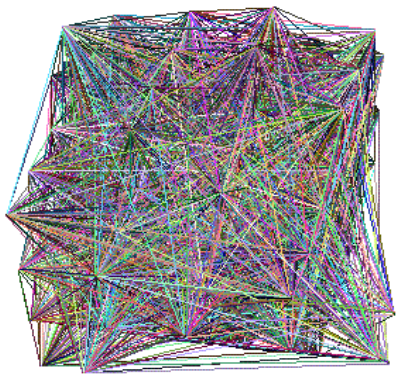

- What does the Internet look like?
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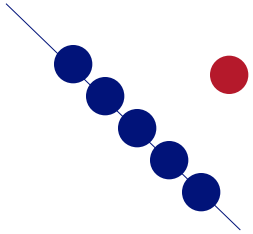
- What is 'normal' / 'abnormal'?
- Which patterns/laws hold?
  - To spot **anomalies** (rarities), we have to discover **patterns**
  - **Large** datasets reveal patterns/anomalies that may be invisible otherwise…



Graph mining

- Are real graphs random?

Laws and patterns

- Are real graphs random?
- A: NO!!
  - Diameter
  - In- and out- degree distributions
  - Other (surprising) patterns

- So, let's look at the data

RUTGERS THE STATE UNIVERSITY OF NEW JERSEY

Carnegie Mellon

Real Graph Patterns

	Unweighted	Weighted
Static	**P01**. Power-law degree distribution [Faloutsos et. al.`99, Kleinberg et. al.`99, Chakrabarti et. al. `04, Newman`04] **P02**. Triangle Power Law [Tsourakakis `08] **P03**. Eigenvalue Power Law [Siganos et. al. `03] **P04**. Community structure [Flake et. al.`02, Girvan and Newman `02] **P05**. Clique Power Laws [Du et. al. '09]	**P12**. Snapshot Power Law [McGlohon et. al. `08]
Dynamic	**P06**. Densification Power Law [Leskovec et. al.`05] **P07**. Small and shrinking diameter [Albert and Barabási `99, Leskovec et. al. '05, McGlohon et. al. '08] **P08**. Gelling point [McGlohon et. al. `08] **P09**. Constant size 2nd and 3rd connected components [McGlohon et. al. `08] **P10**. Principal Eigenvalue Power Law [Akoglu et. al. `08] **P11**. Bursty/self-similar edge/weight additions [Gomez and Santonja `98, Gribble et. al. `98, Crovella and Bestavros `99, McGlohon et .al. `08]	**P13**. Weight Power Law [McGlohon et. al. `08] **P14**. Skewed call duration distributions [Vaz de Melo et. al. `10]

RTG: A Recursive Realistic Graph Generator using Random Typing

Leman Akoglu and Christos Faloutsos. *ECML PKDD*'09.

Roadmap

- Patterns in graphs
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  - Weighted graphs
  - Time-evolving graphs
- Anomaly Detection
- Application: ebay fraud
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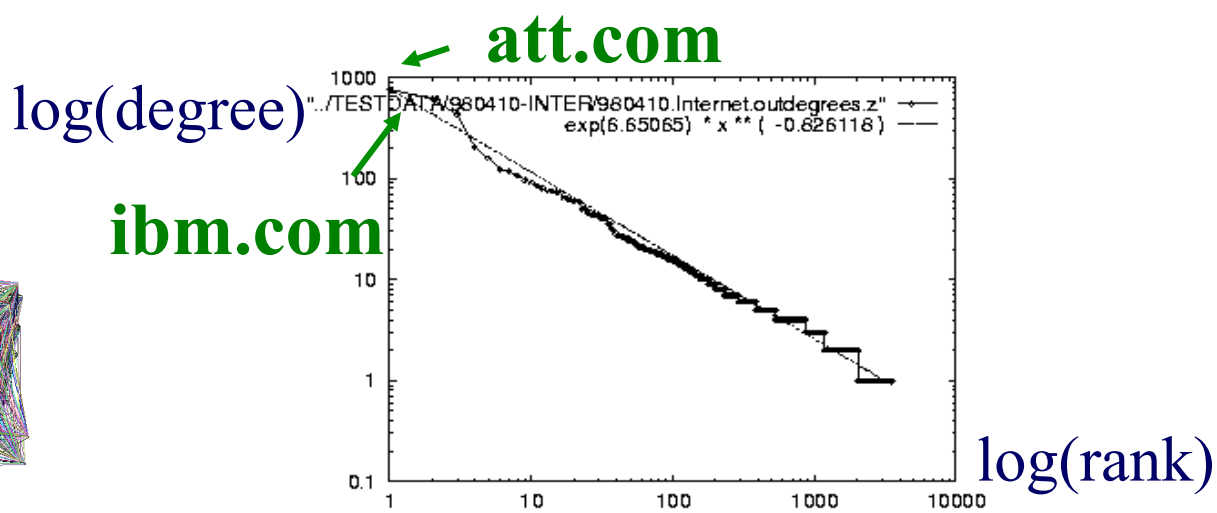
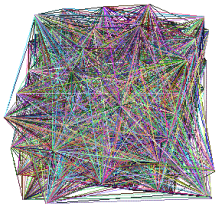
Solution# S.1

- Power law in the degree distribution [SIGCOMM99]

Internet Domains





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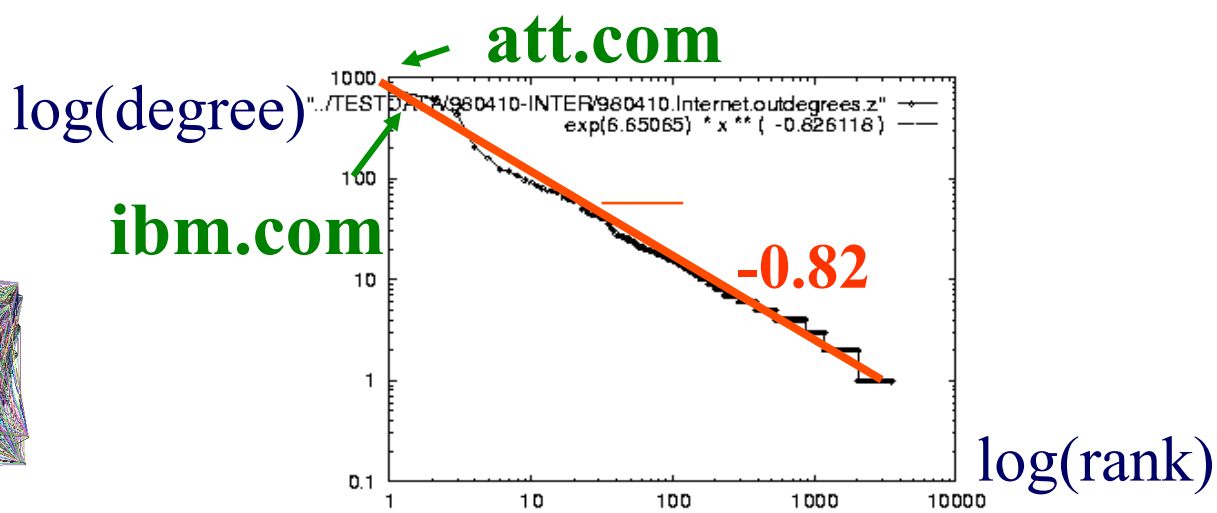
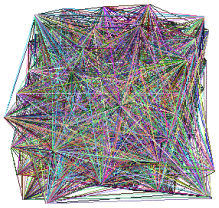
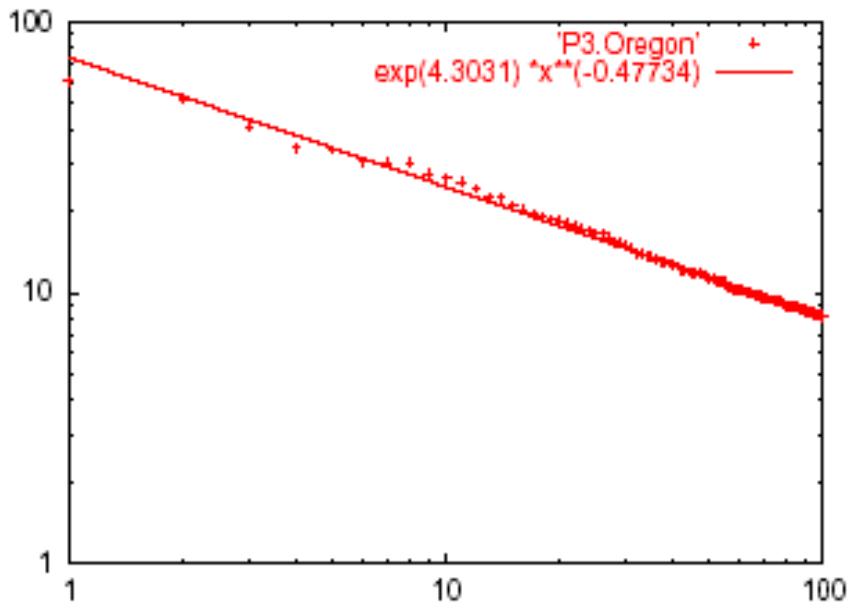




Solution# S.2: Eigen Exponent *E*

Eigenvalue



Rank of decreasing eigenvalue

Exponent = slope

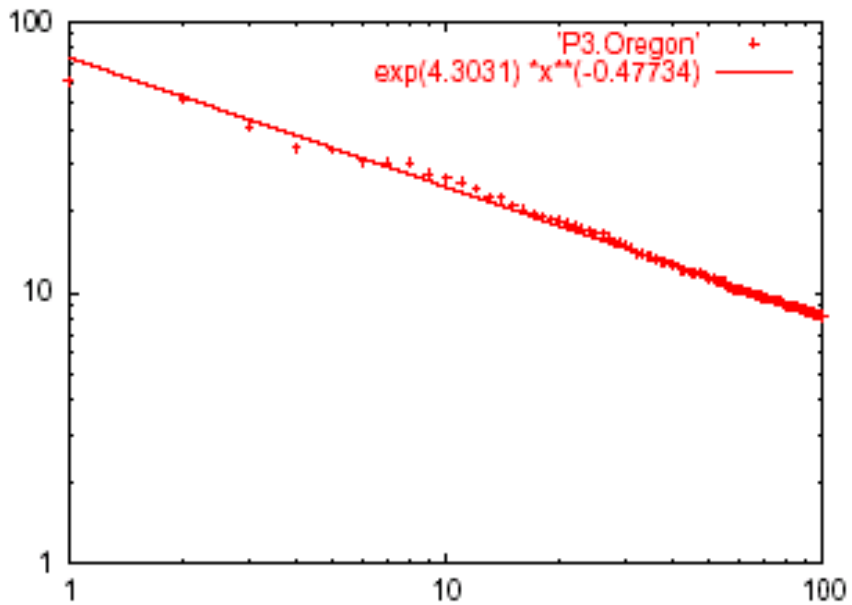
E = -0.48

May 2001

- A2: power law in the eigenvalues of the adjacency matrix

Solution# S.2: Eigen Exponent *E*

Eigenvalue



Rank of decreasing eigenvalue

Exponent = slope

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May 2001

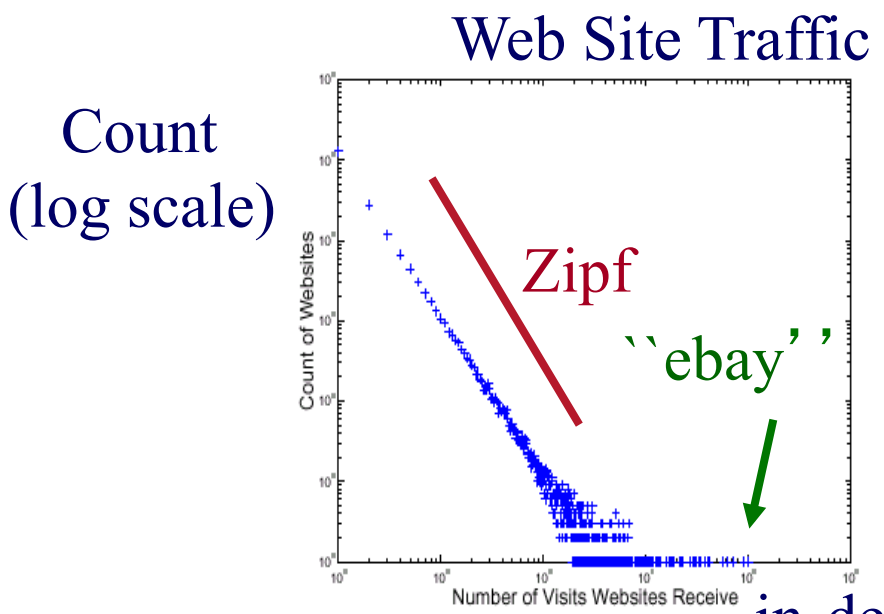
- [Mihail, Papadimitriou '02]: slope is ½ of rank exponent

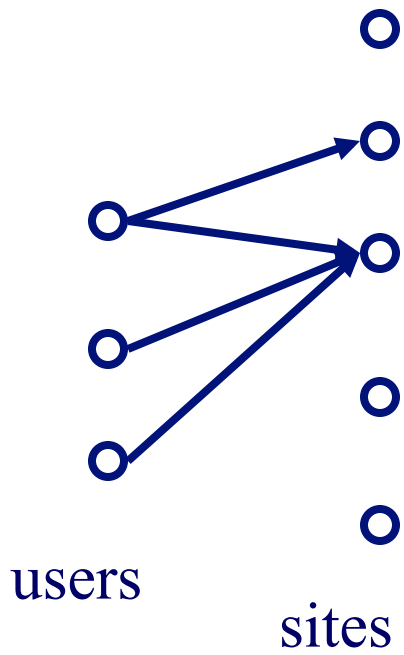
But:

How about graphs from other domains?

More power laws:

- web hit counts [w/ A. Montgomery]

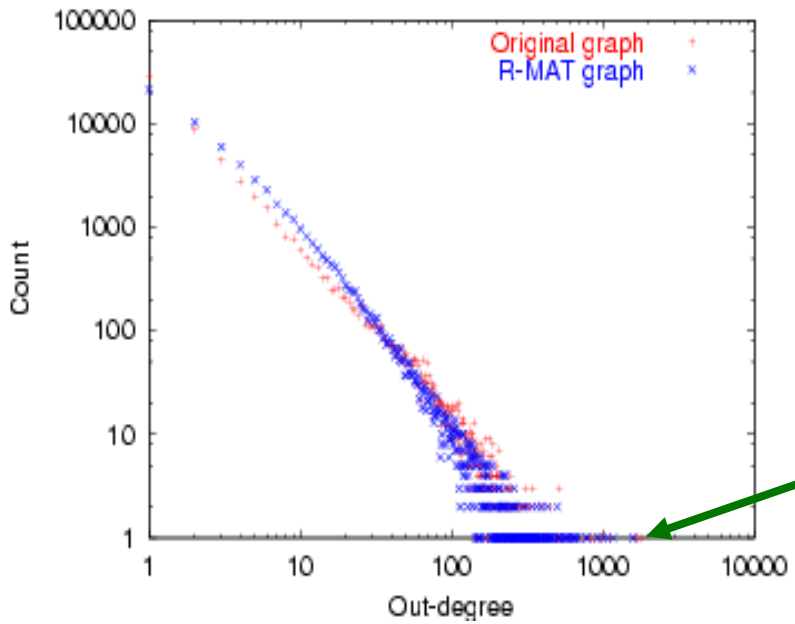




epinions.com

- who-trusts-whom [Richardson + Domingos, KDD 2001]

count



trusts-2000-people user

(out) degree

And numerous more

- # of sexual contacts
- Income [Pareto] – '80-20 distribution'
- Duration of downloads [Bestavros+]
- Duration of UNIX jobs (`mice and elephants')
- Size of files of a user
- …
- 'Black swans'

Roadmap

- Patterns in graphs
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  - Static graphs
    - S1: Degree, S2: Eigenvalues
    - S3-4: Triangles, S5: Cliques
    - Radius plot
    - Other observations ('EigenSpokes')
  - Weighted graphs
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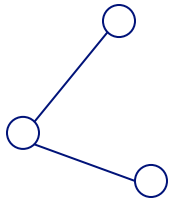
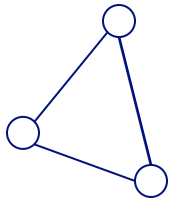
Solution# S.3: Triangle 'Laws'



- Real social networks have a lot of triangles

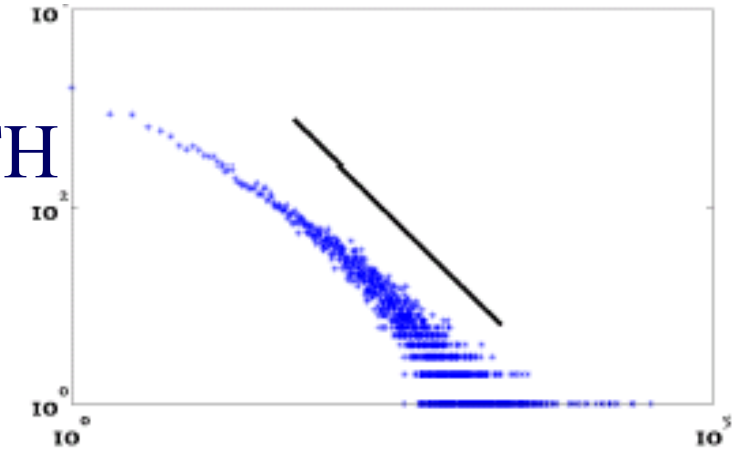
Solution# S.3: Triangle 'Laws'



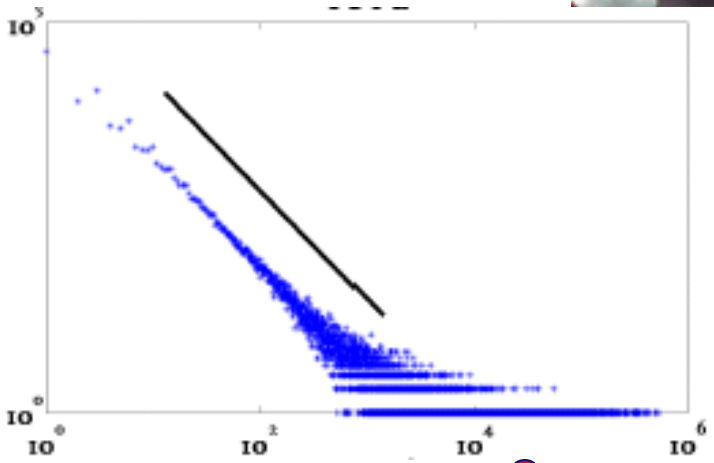
- Real social networks have a lot of triangles
  - Friends of friends are friends
- Any patterns?

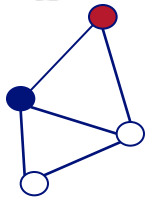
Triangle Law: #S.3
[Tsourakakis ICDM 2008]

HEP-TH

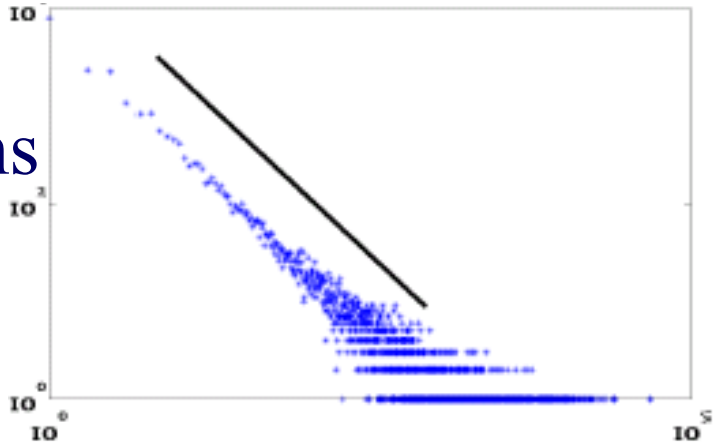


ASN





Epinions



X-axis: # of participating triangles

Y: count (~ pdf)

Triangle Law: #S.3
[Tsourakakis ICDM 2008]

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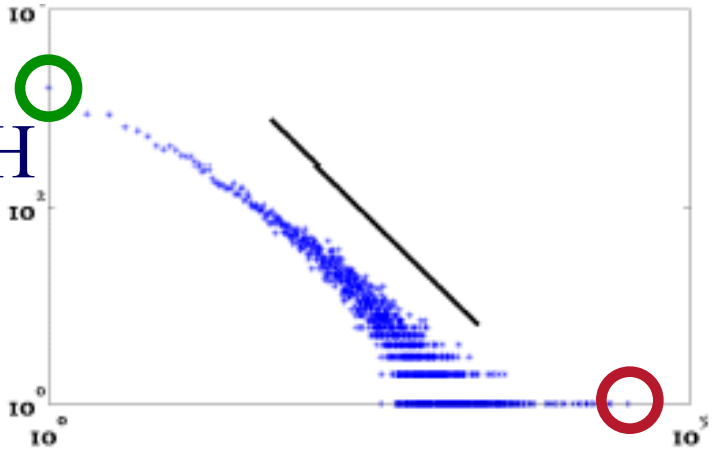

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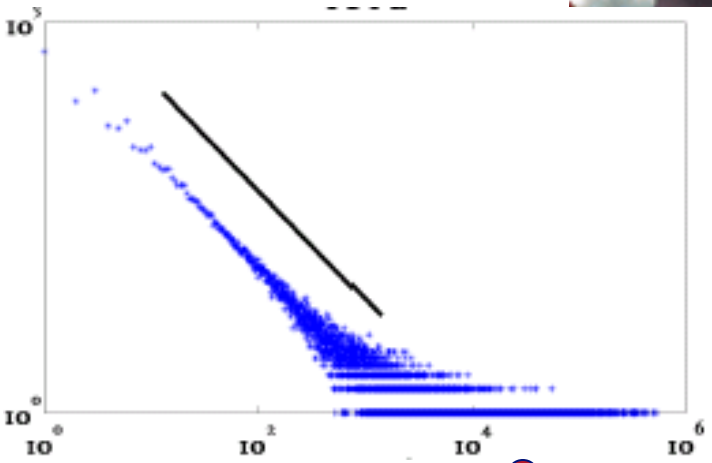

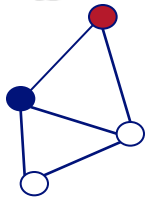

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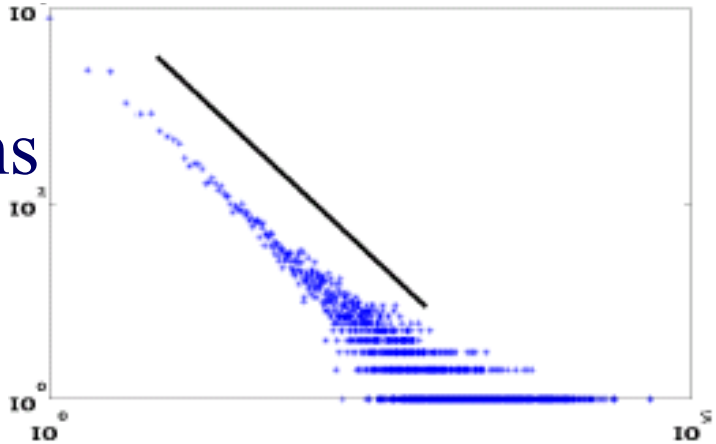

X-axis: # of participating triangles

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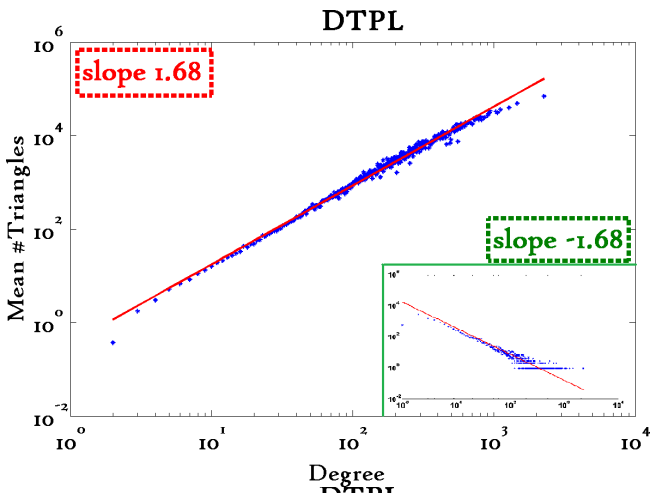
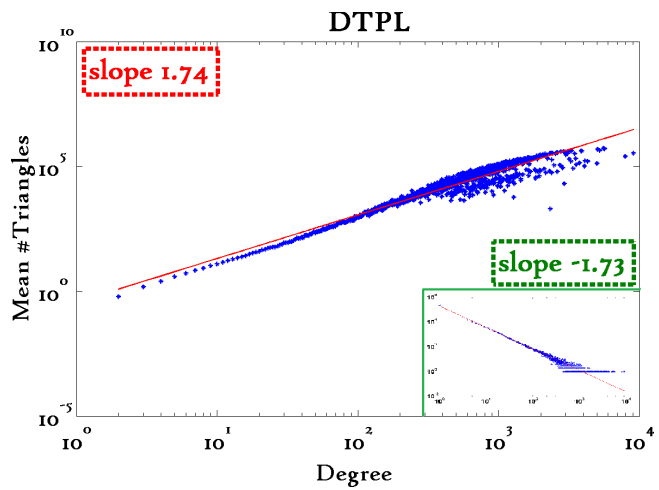
Triangle Law: #S.4
[Tsourakakis ICDM 2008]

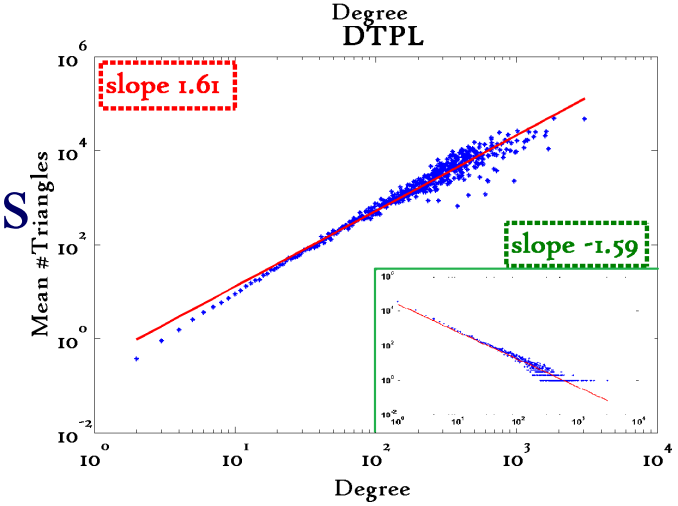
Reuters





SN

Epinions



X-axis: degree

Y-axis: mean # triangles

n friends $\rightarrow \sim n^{1.6}$ triangles

Triangle Law: Computations
[Tsourakakis ICDM 2008]

Triangles are expensive to compute

(3-way join; several approximation algorithms)

Q: Can we do this computation quickly?

Triangle Law: Computations
[Tsourakakis ICDM 2008]

Triangle are expensive to compute

(3-way join; several approx. algos)

Q: Can we do this computation quickly?

A: Yes!

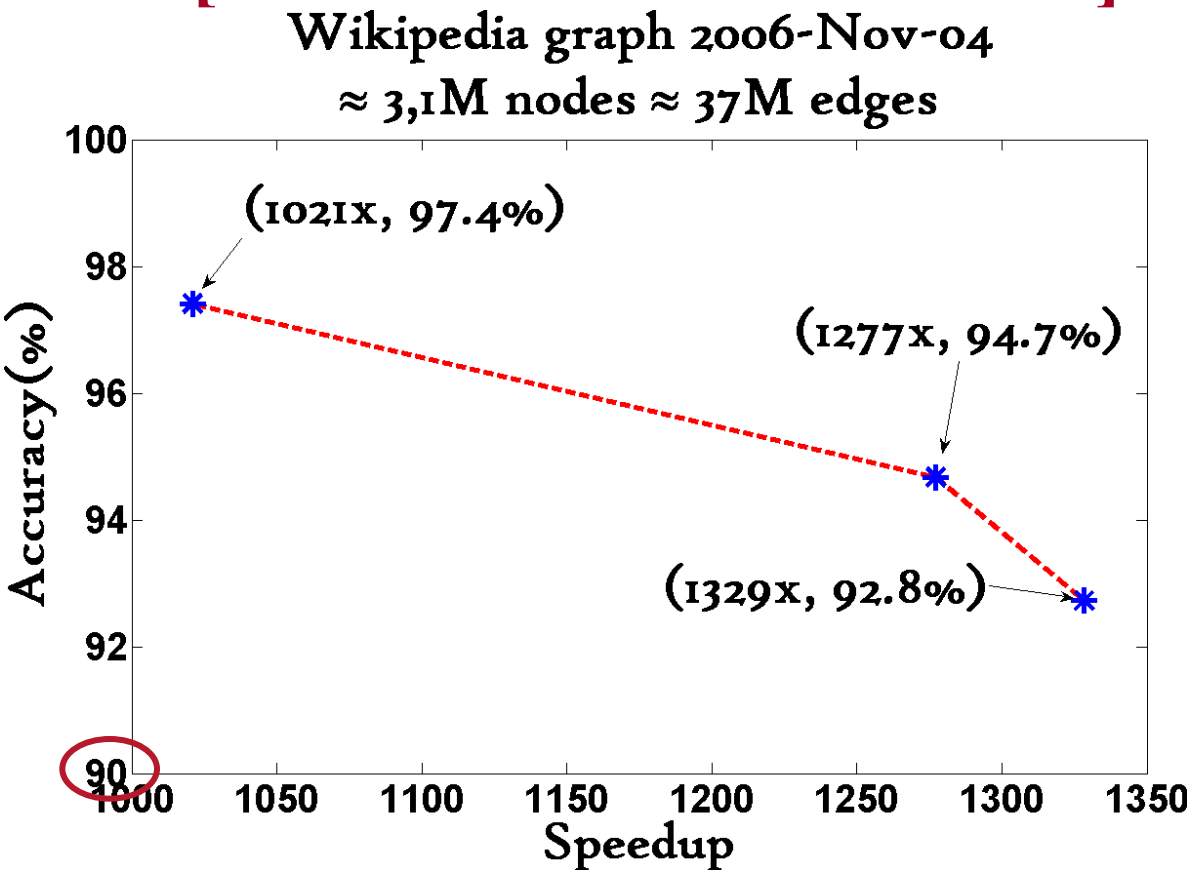
#triangles = 1/6 Sum (λ_i^3)

(and, because of skewness (S2) ,

we only need the top few eigenvalues!)

Triangle Law: Computations

[Tsourakakis ICDM 2008]



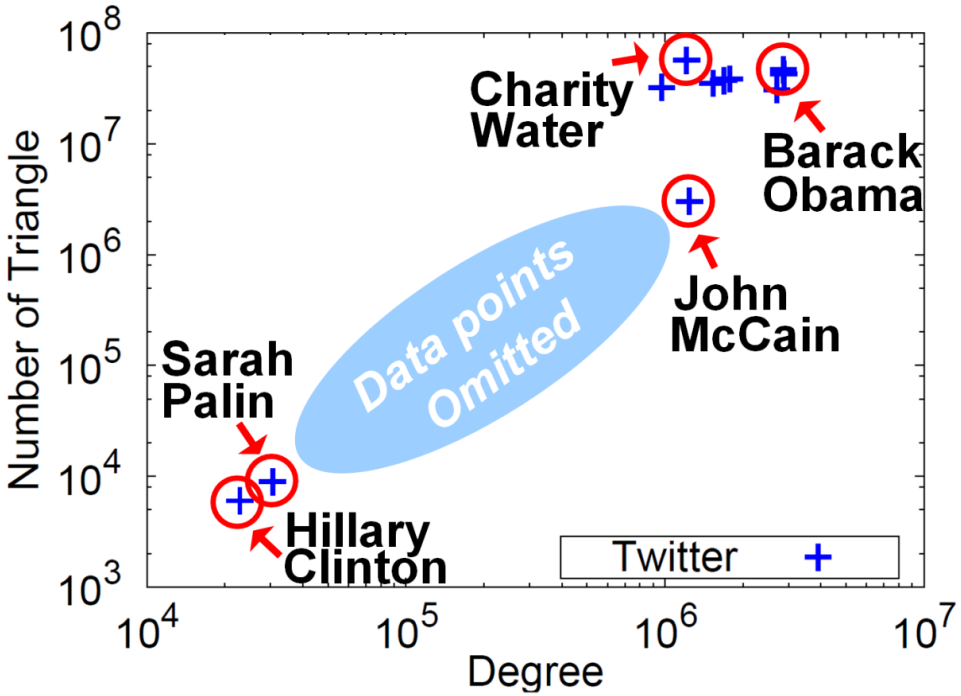
1000x+ speed-up, >90% accuracy

Triangle Counting on Big Graphs

Anomalous nodes in Twitter (~ 3 billion edges)

[U Kang, Brendan Meeder, +, PAKDD'11]

Triangle Counting on Big Graphs



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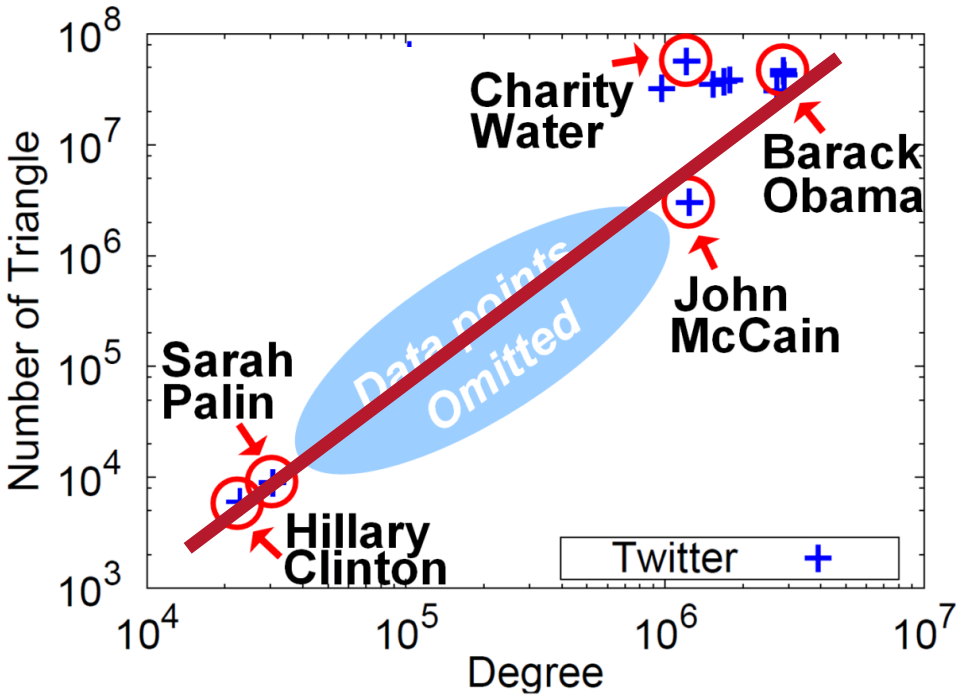

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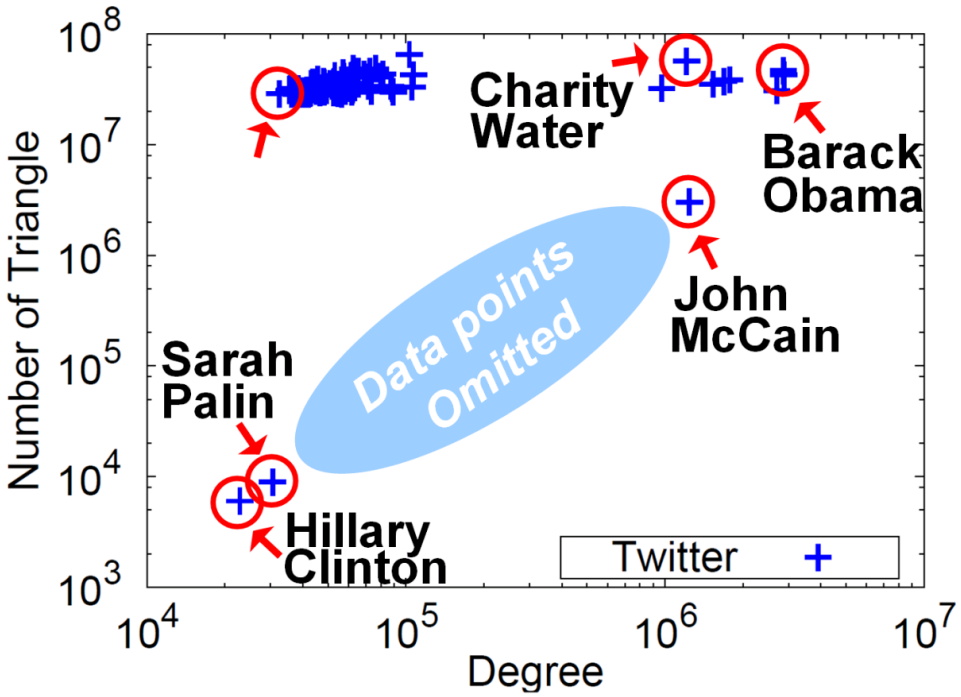

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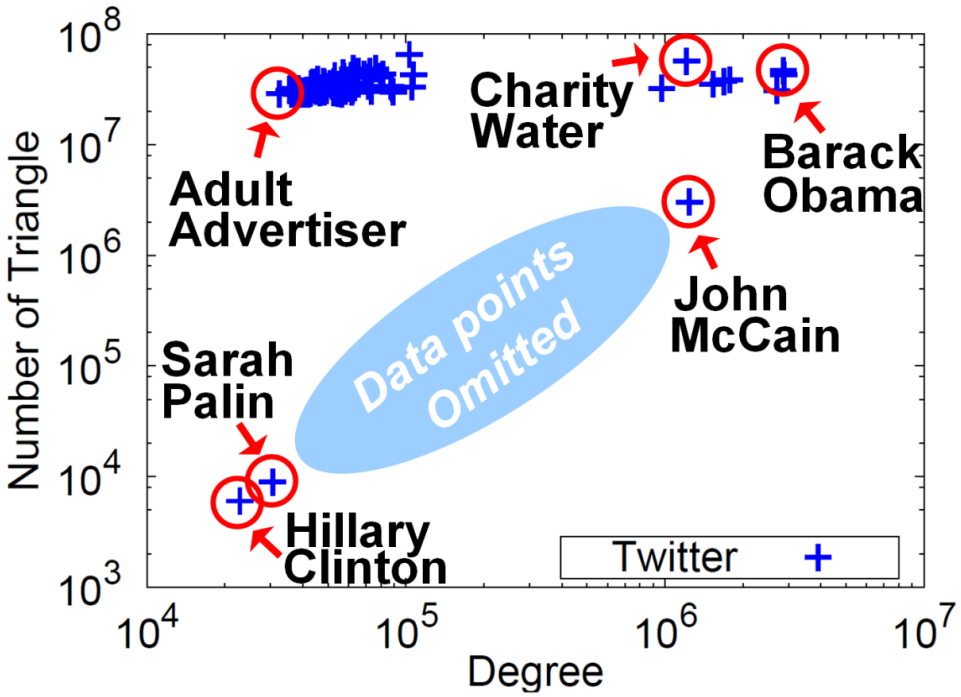

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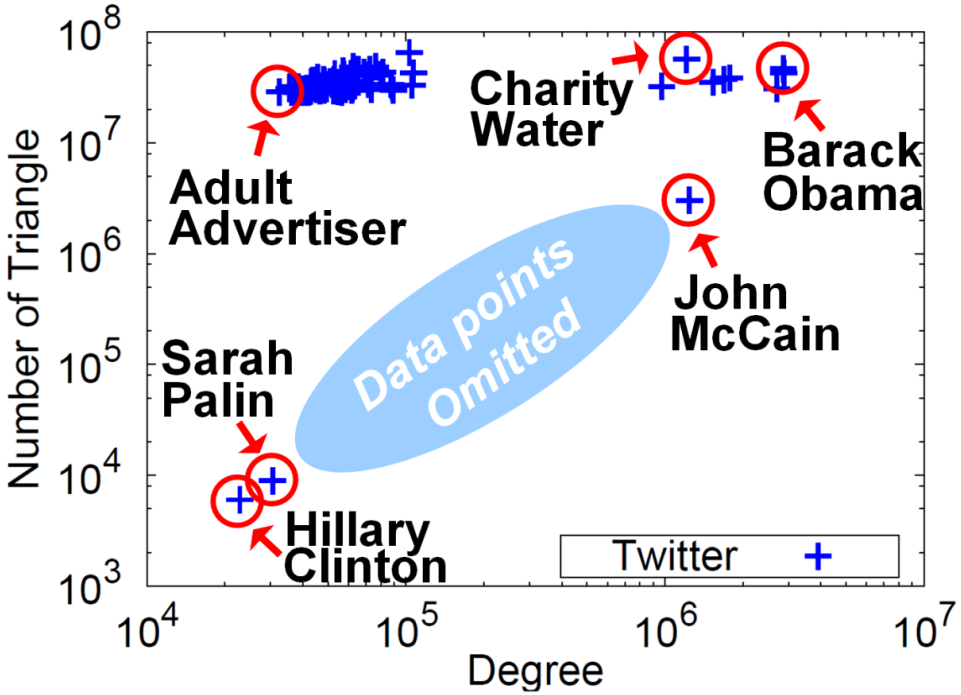
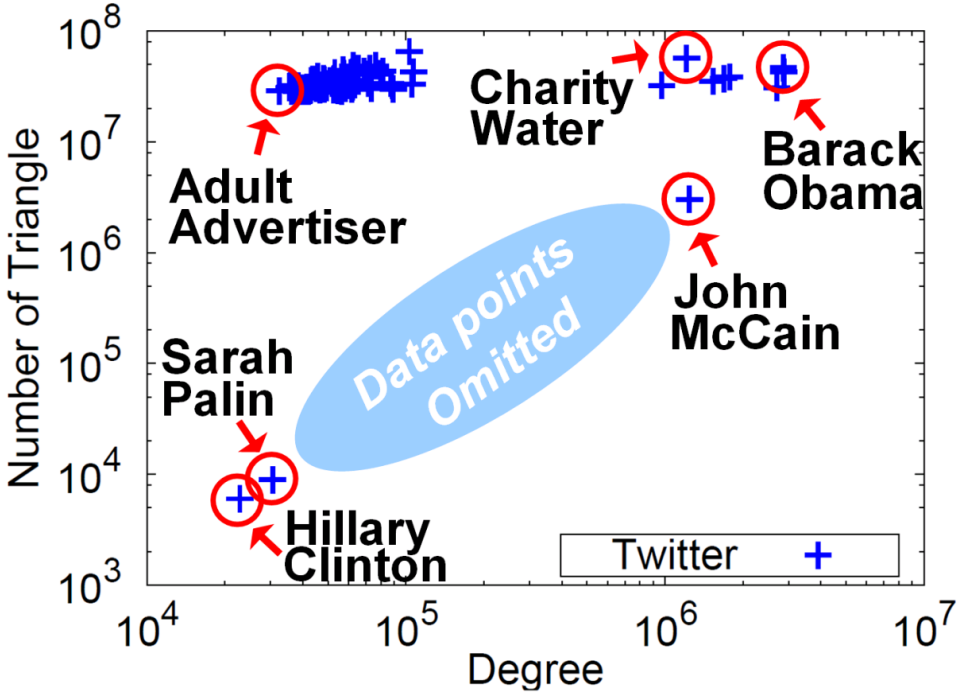


Q: How to compute # triangles in B-node graph? ($O(d_{max}^2)$)?

Triangle Counting on Big Graphs



Q: How to compute # triangles in B-node graph? **A: cubes of eigvals**

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    - S1: Degree, S2: Eigenvalues
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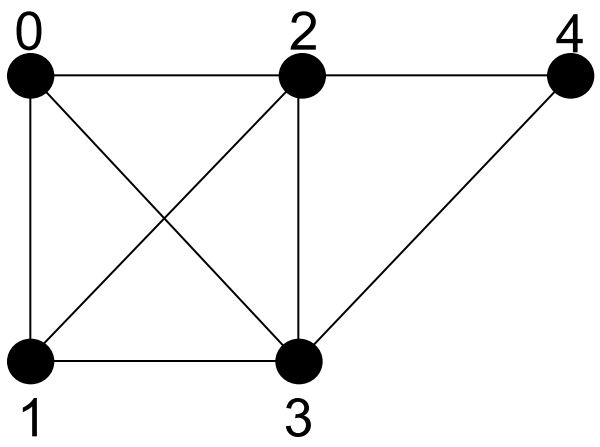
How about cliques?

- *Large Human Communication Networks Patterns and a Utility-Driven Generator*. **Nan Du**, Christos Faloutsos, Bai Wang, Leman Akoglu, KDD 2009

Cliques

- Clique is a complete subgraph.
- If a clique can not be contained by any larger clique, it is called the maximal clique.



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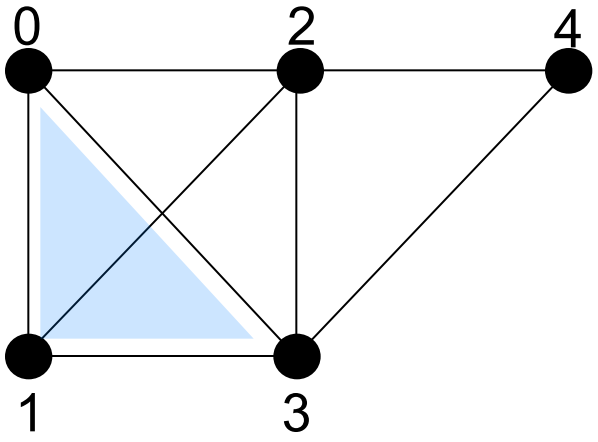

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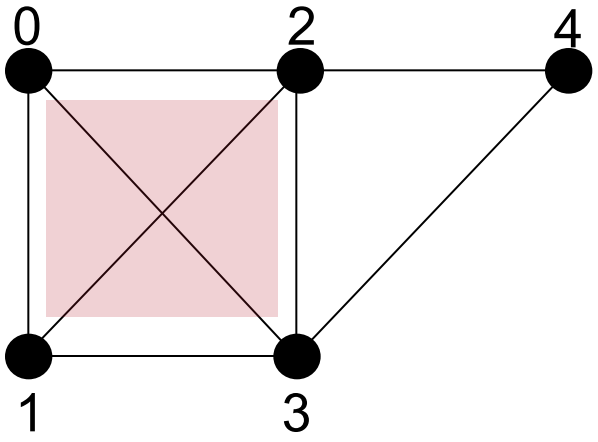
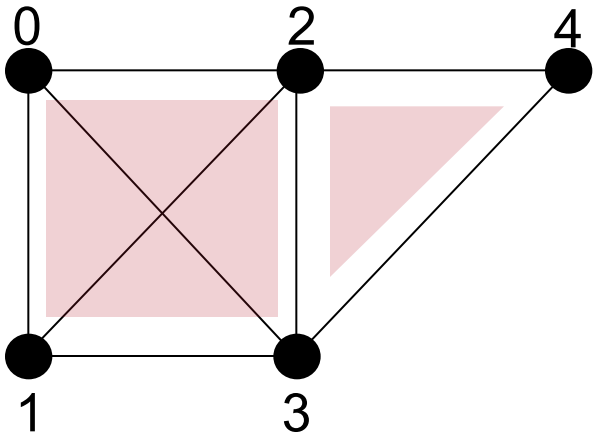


Clique

- Clique is a complete subgraph.
- If a clique can not be contained by any larger clique, it is called the maximal clique.
- {0,1,2}, {0,1,3}, {1,2,3} {2,3,4}, {0,1,2,3} are cliques;
- {**0,1,2,3**} and {**2,3,4**} are the maximal cliques.



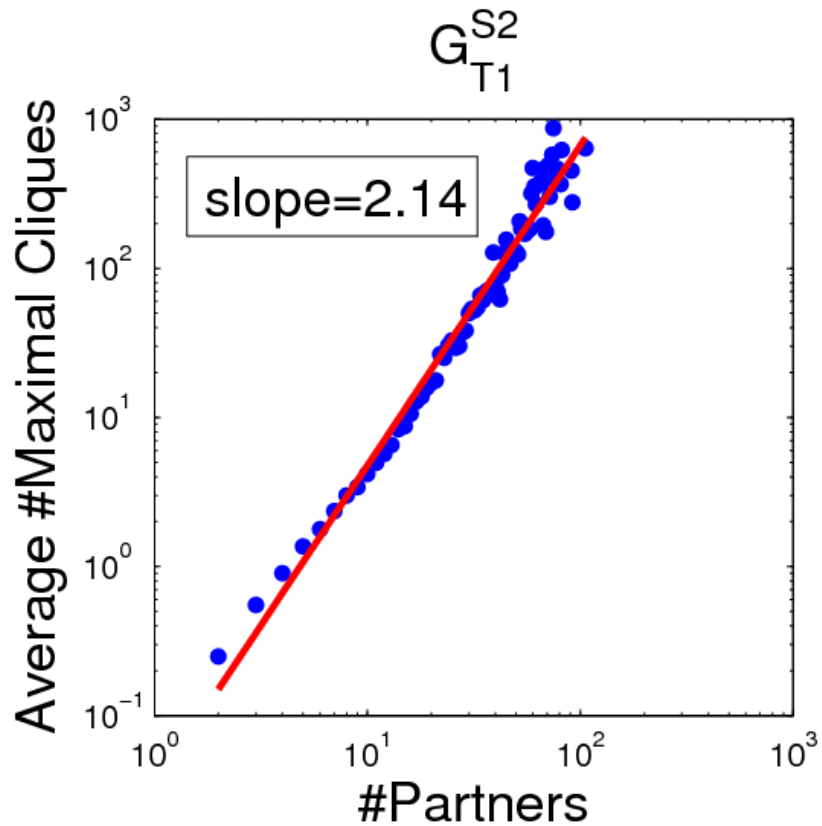
S5: Clique-Degree Power-Law

- Power law:

$$C_{avg}^{d_i} \propto d_i^{\alpha}$$

maximal cliques of node i

degree of node i

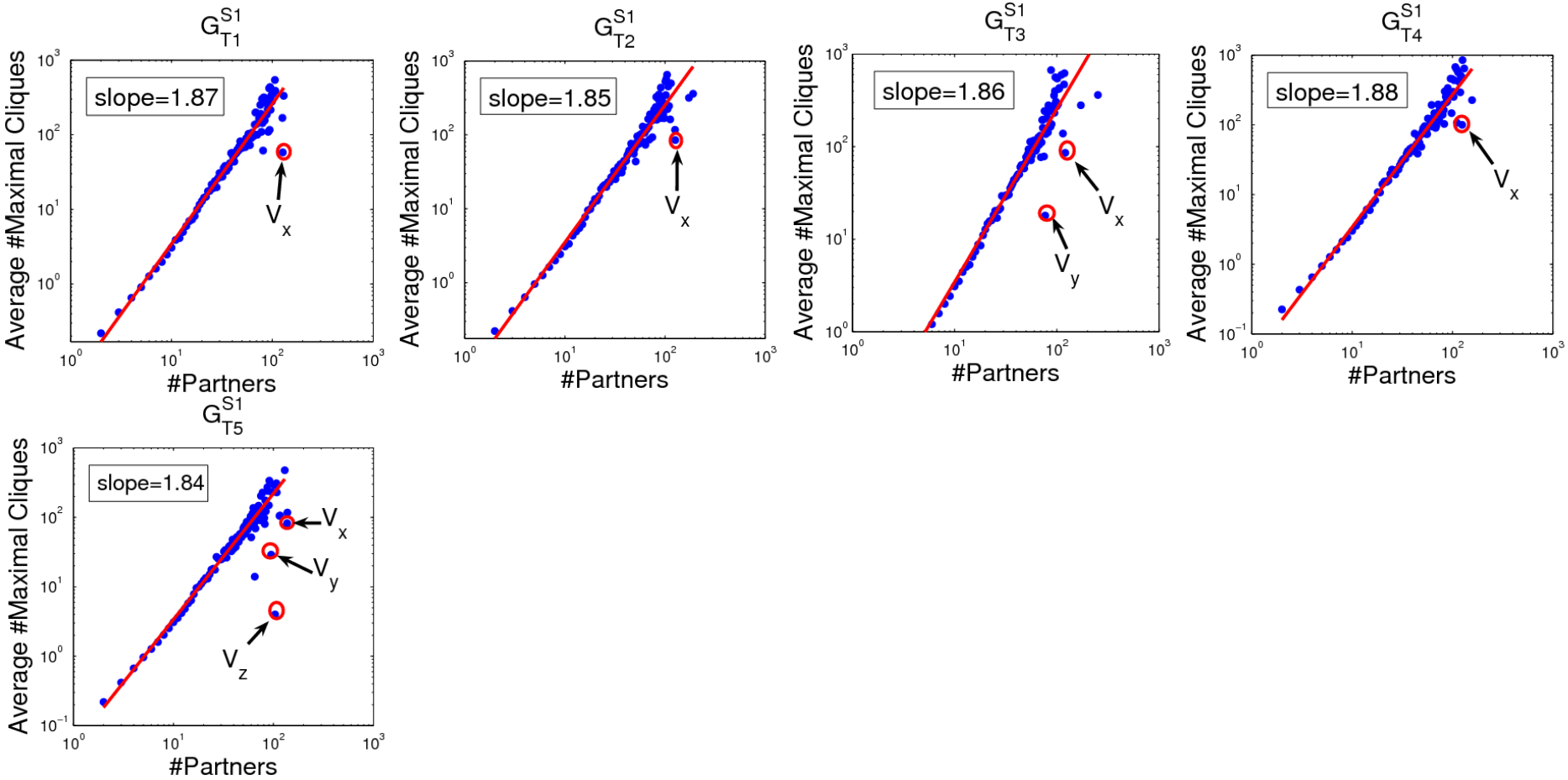


α is the power law exponent
$\alpha \in [1.8, 2.2]$ for S1~S3

More friends, even more social circles !

S5: Clique-Degree Power-Law

- Outlier Detection



S5: Clique-Degree Power-Law

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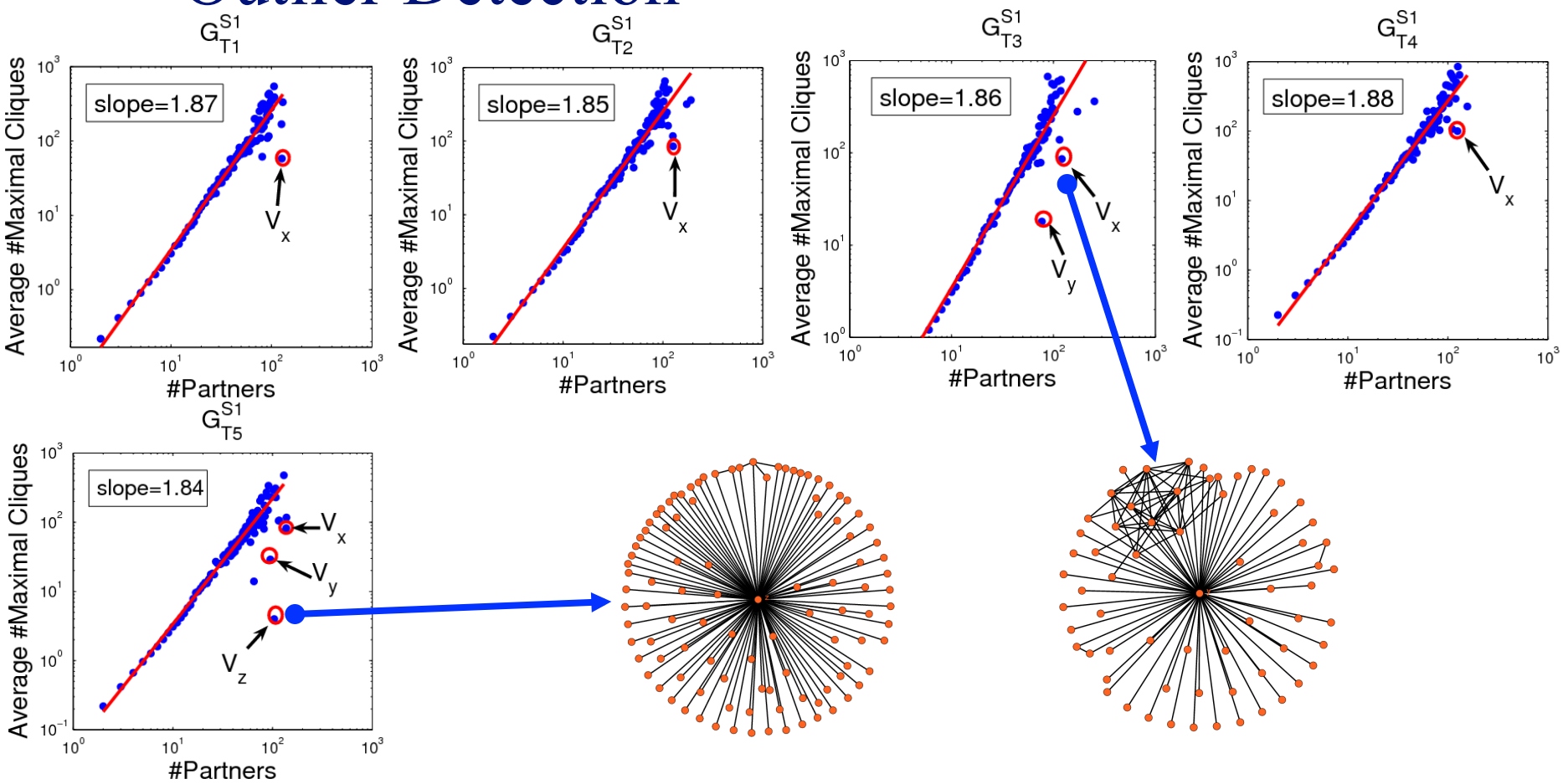

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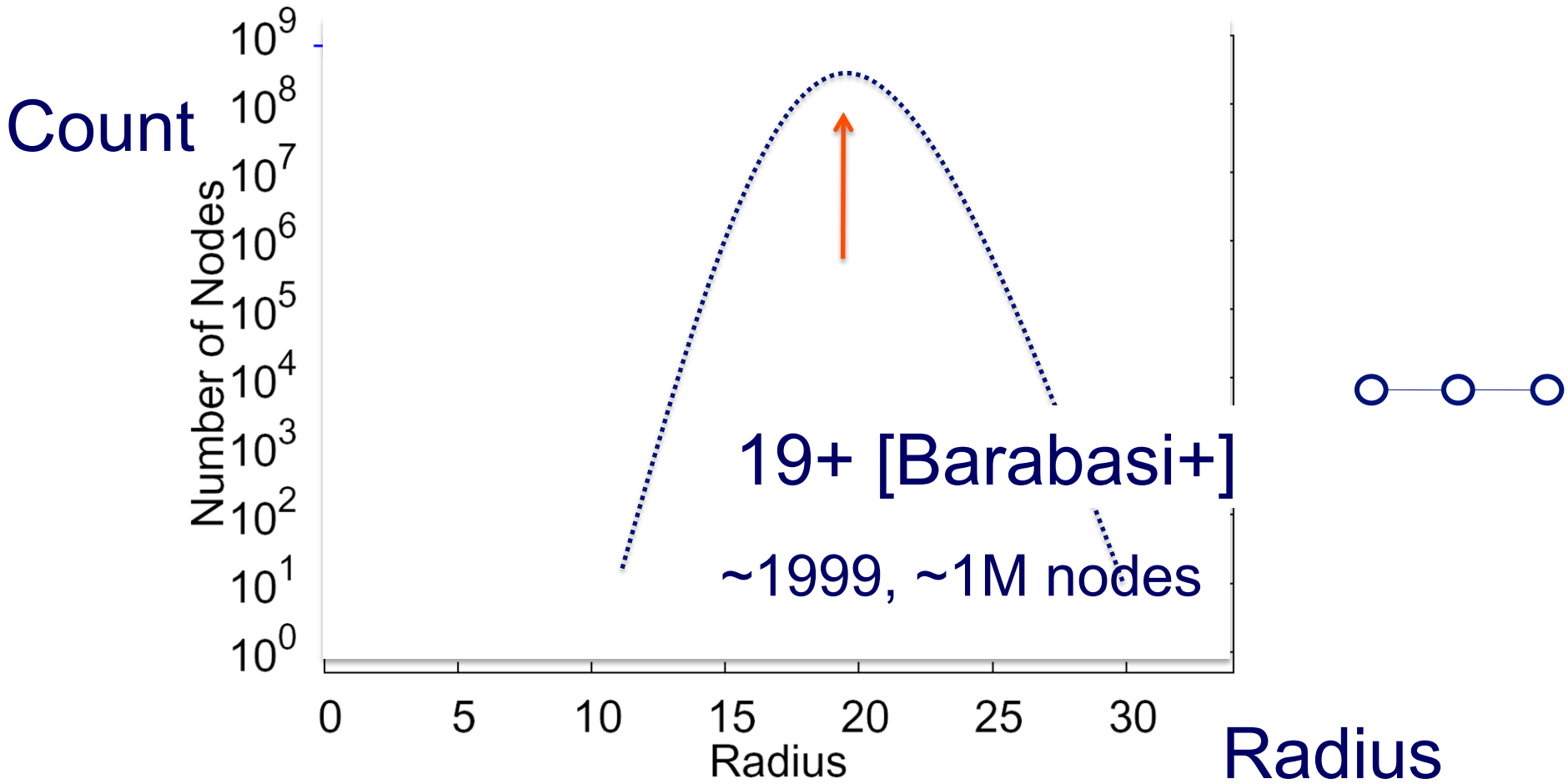
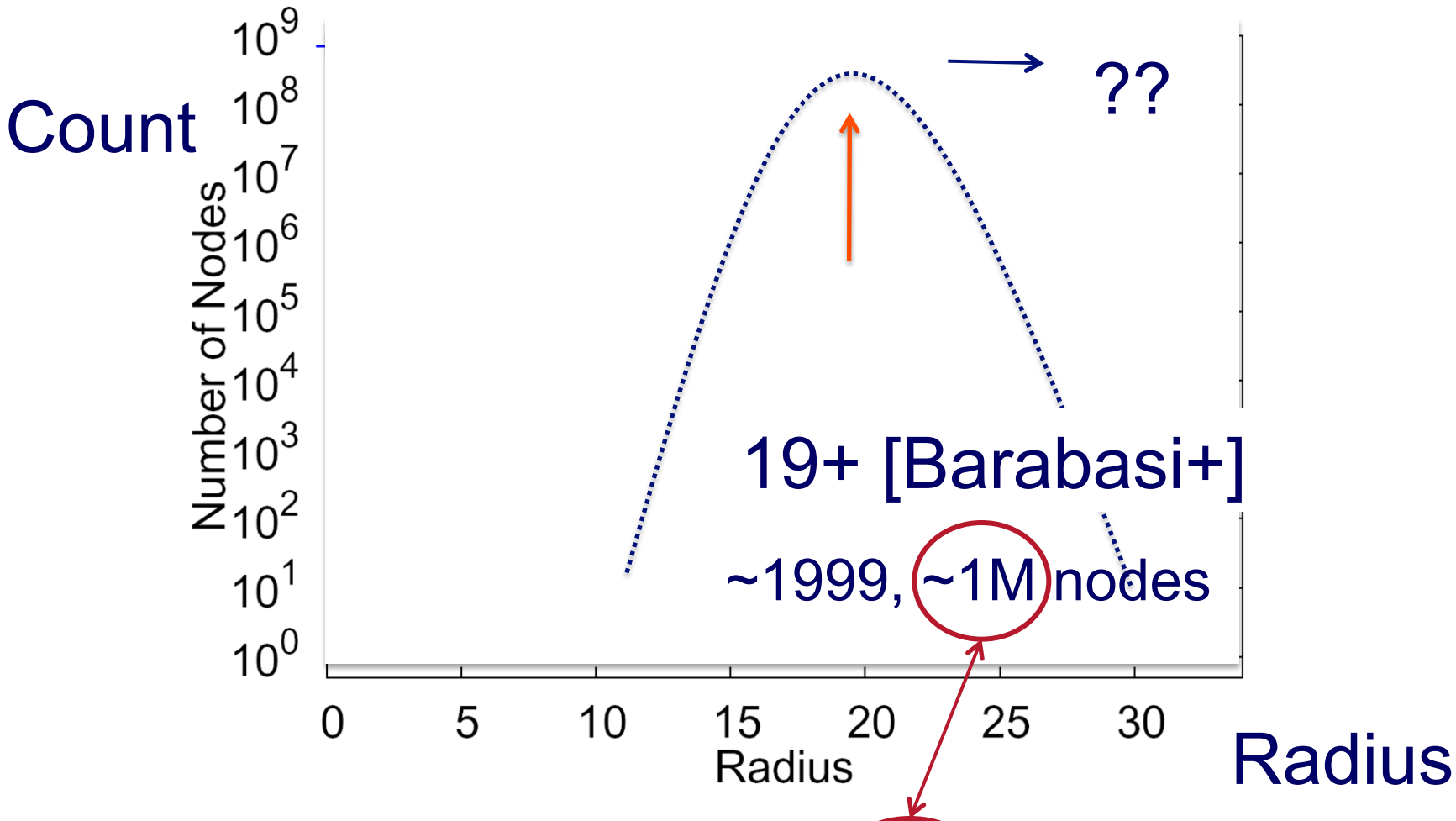
    - Radius plot
    - Other observations ('EigenSpokes')
  - Weighted graphs
  - Time-evolving graphs

HADI for Diameter Estimation

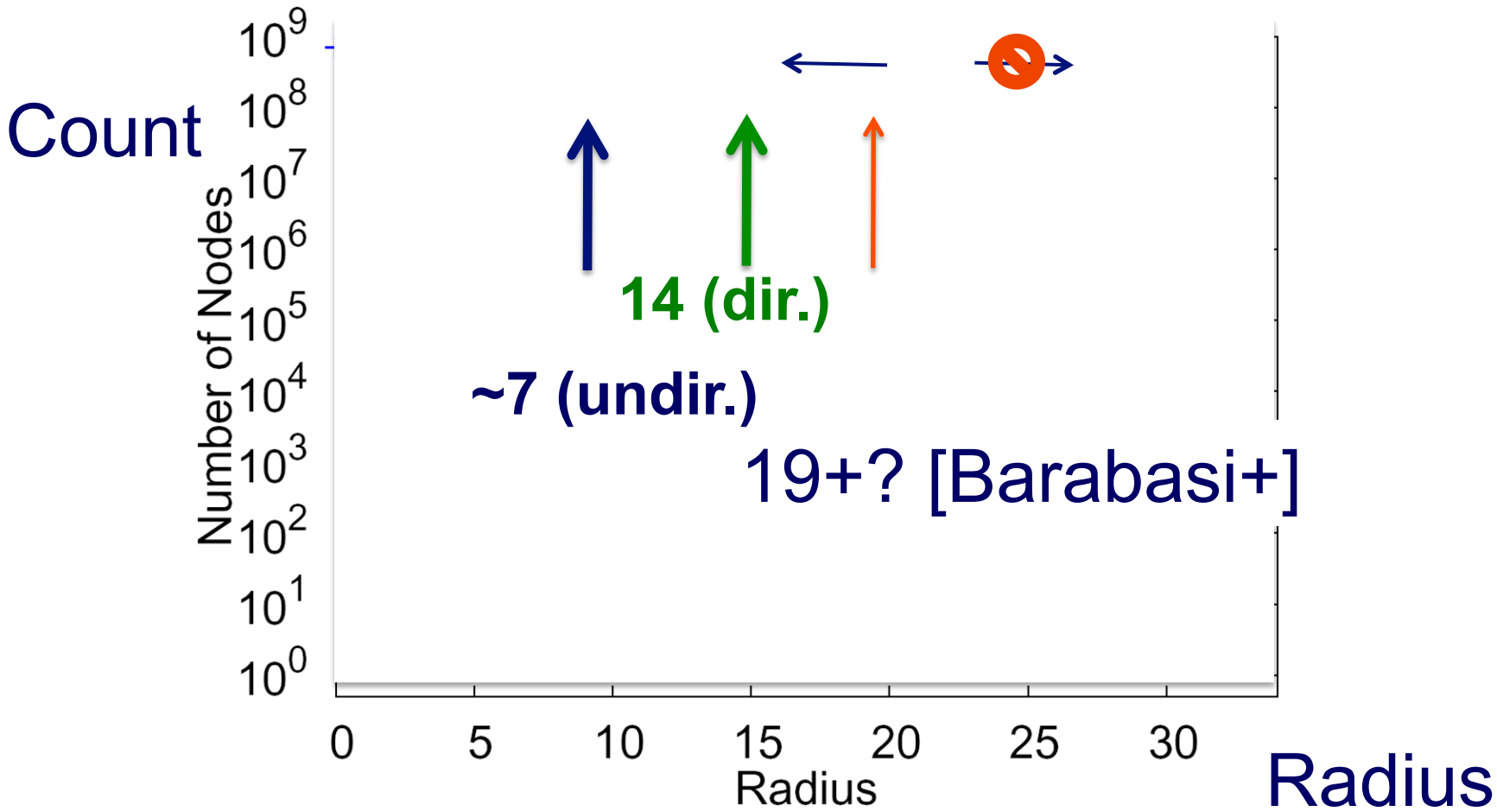
- *Radius Plots for Mining Tera-byte Scale Graphs* **U Kang**, Charalampos Tsourakakis, Ana Paula Appel, Christos Faloutsos, Jure Leskovec, SDM'10
- Naively: diameter needs $O(N^2)$ space and up to $O(N^3)$ time – **prohibitive** (N~1B)
- Our HADI: linear on E (~10B)
  - Near-linear scalability w.r.t. # machines
  - Several optimizations → 5x faster


Count
Number of Nodes
10^9
10^8
10^7
10^6
10^5
10^4
10^3
10^2
10^1
10^0
0
5
10
15
20
25
30
Radius
19+ [Barabasi+]
~1999, ~1M nodes
Radius



YahooWeb graph (120Gb, 1.4B nodes, 6.6 B edges)

- Largest publicly available graph ever studied.



YahooWeb graph (120Gb, 1.4B nodes, 6.6 B edges)

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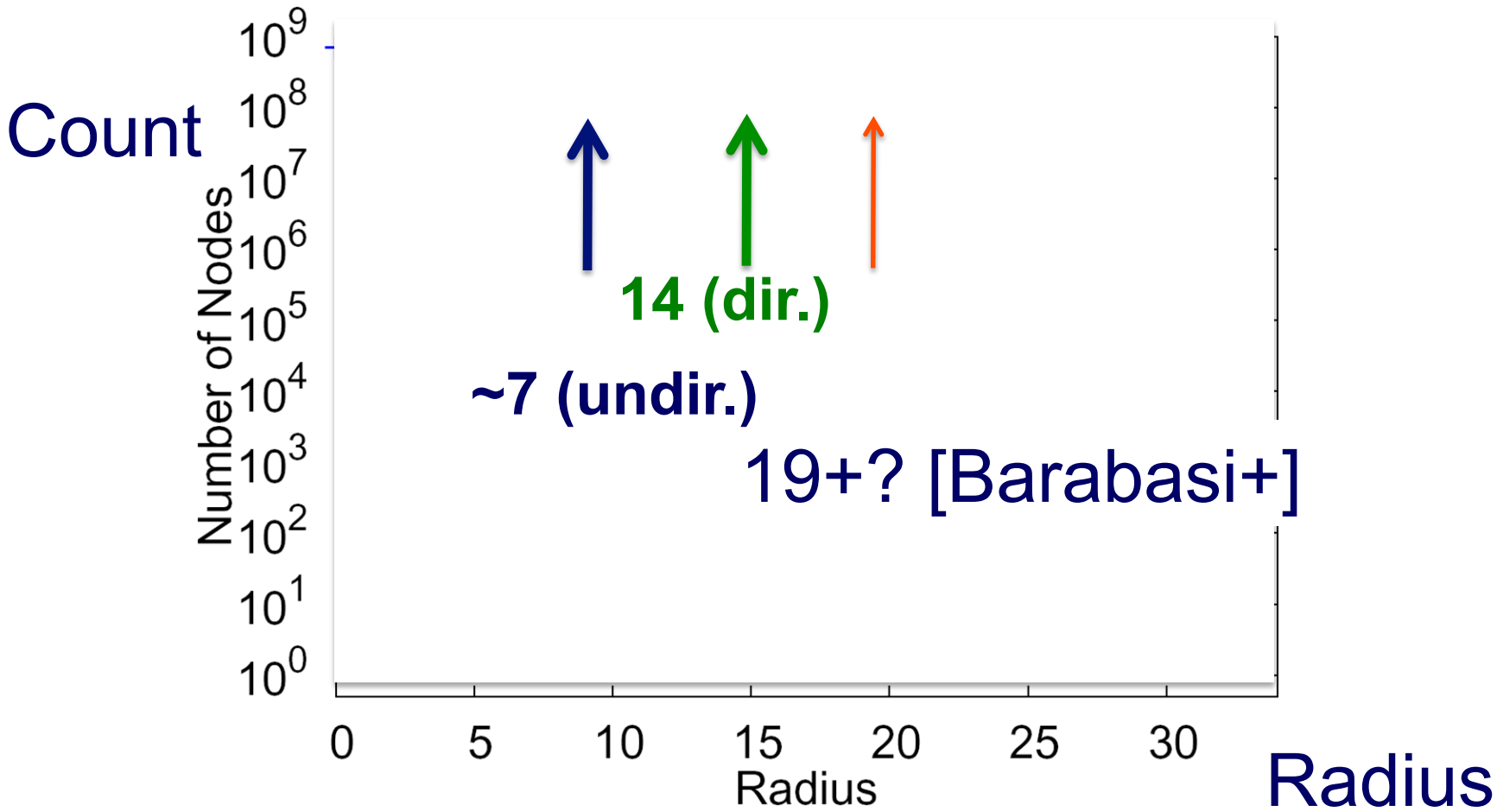
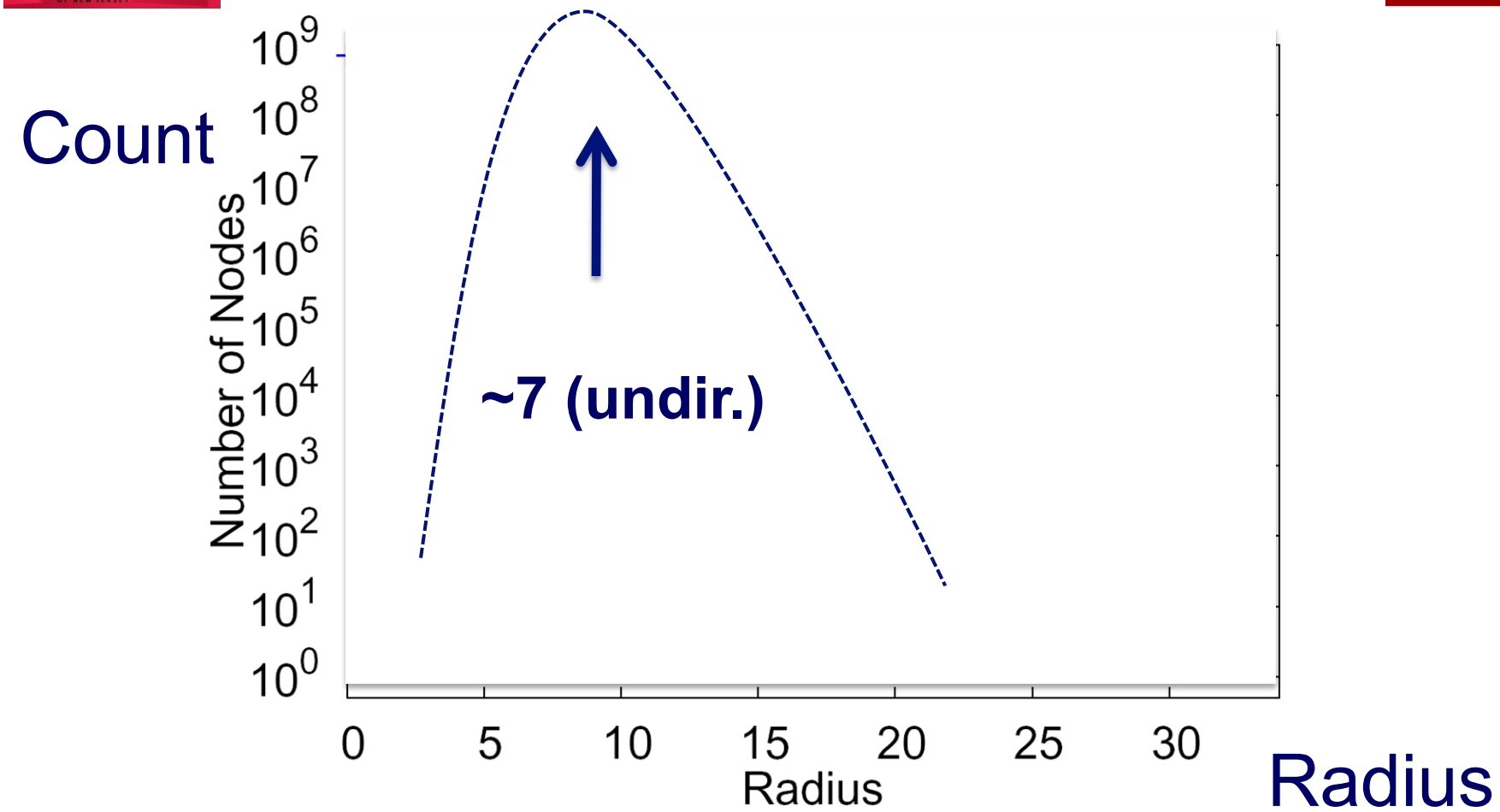


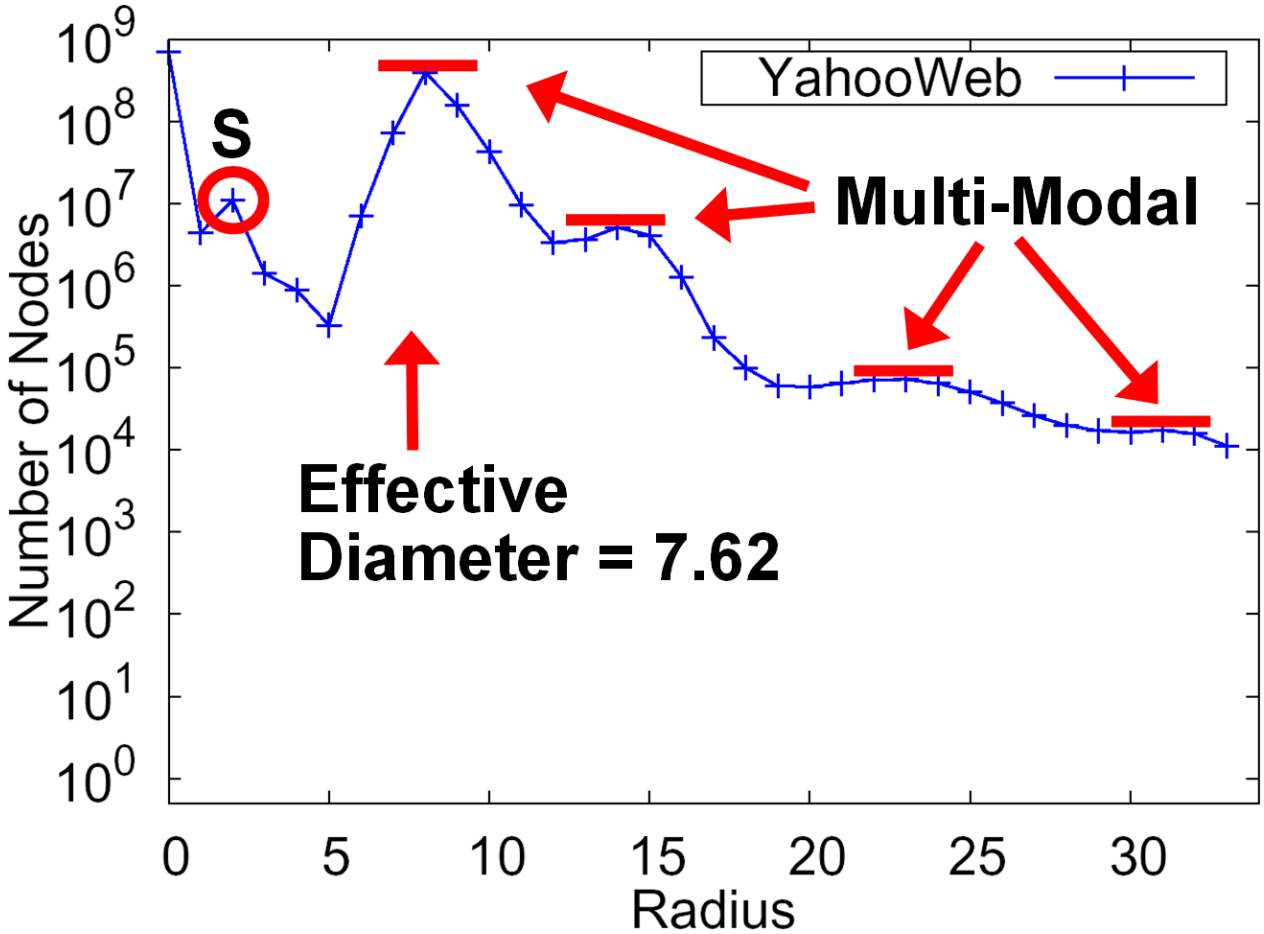
YahooWeb graph (120Gb, 1.4B nodes, 6.6 B edges)

- 7 degrees of separation (!)
- Diameter: shrunk



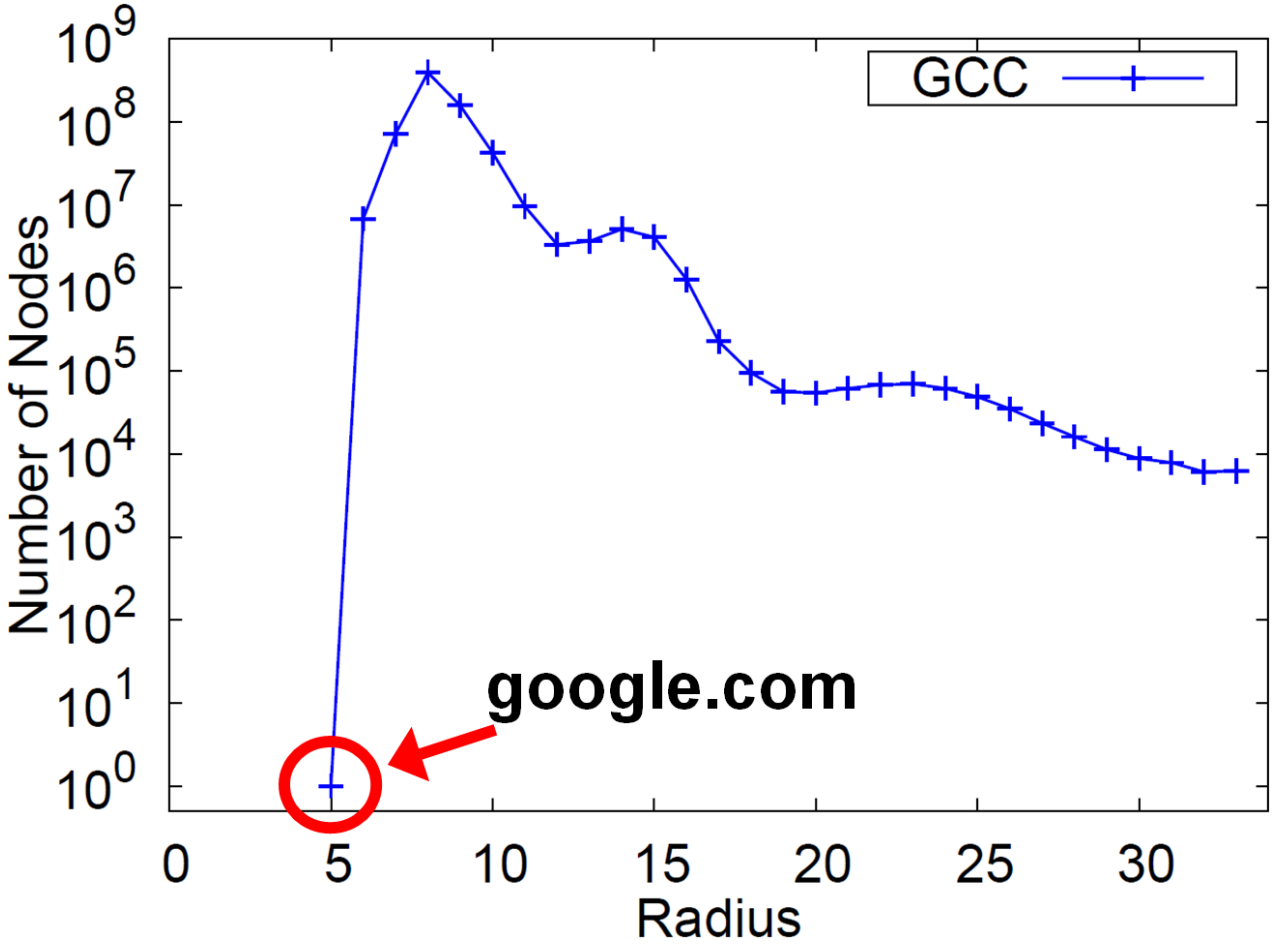
YahooWeb graph (120Gb, 1.4B nodes, 6.6 B edges)

Q: Shape?

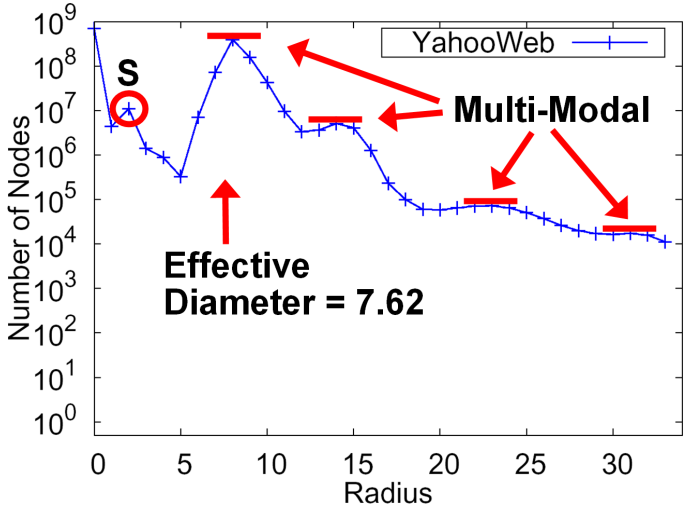


YahooWeb graph (120Gb, 1.4B nodes, 6.6 B edges)

- Effective diameter: surprisingly small
- Multi-modality (?!)

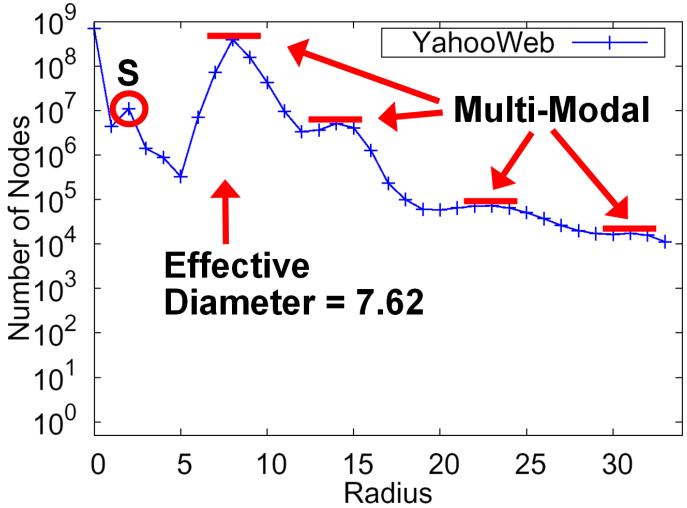


Radius Plot of **GCC** of YahooWeb Graph

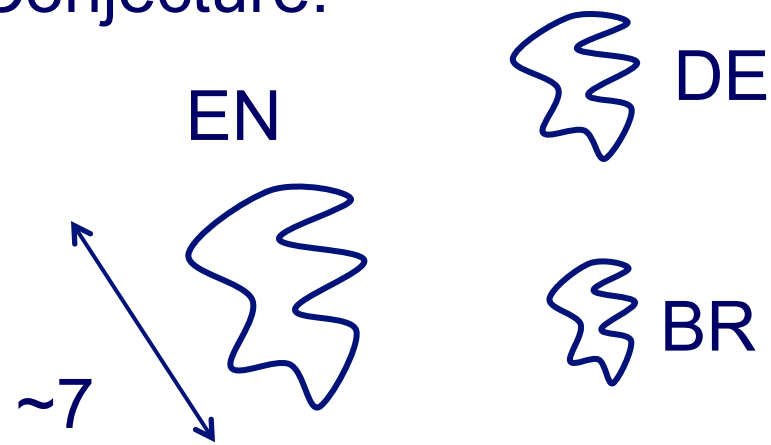


YahooWeb graph (120Gb, 1.4B nodes, 6.6 B edges)

- Effective diameter: surprisingly small
- Multi-modality: probably mixture of cores



Conjecture:



YahooWeb graph (120Gb, 1.4B nodes, 6.6 B edges)

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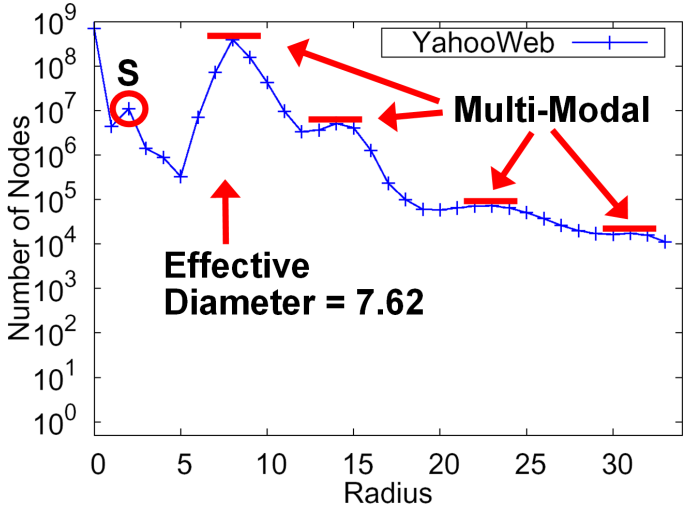
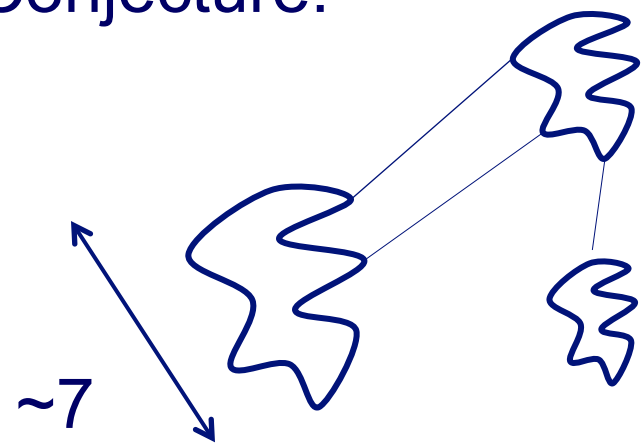


Conjecture:



YahooWeb graph (120Gb, 1.4B nodes, 6.6 B edges)

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Roadmap

- Patterns in graphs
  - overview
  - Static graphs
    - S1: Degree, S2: Eigenvalues
    - S3-4: Triangles, S5: Cliques
    - Radius plot
    - Other observations ('EigenSpokes')
  - Weighted graphs
  - Time-evolving graphs

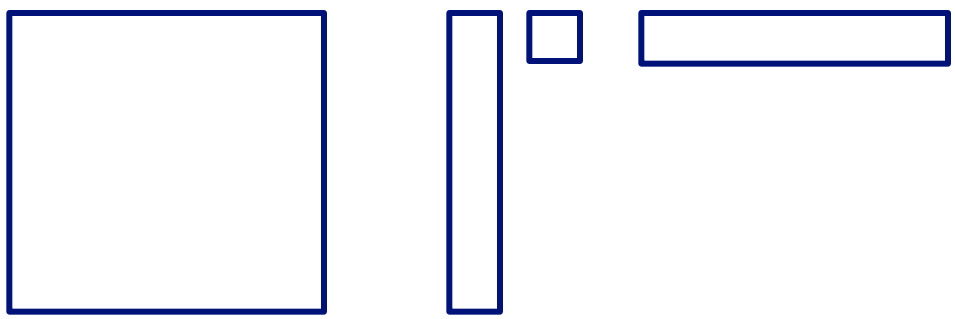
S6: EigenSpokes

EigenSpokes: Surprising Patterns and Scalable Community Chipping in Large Graphs.
B. Aditya Prakash, Mukund Seshadri, Ashwin Sridharan, Sridhar Machiraju, and Christos Faloutsos: PAKDD 2010.

EigenSpokes

- Eigenvectors of adjacency matrix
  - equivalent to singular vectors (symmetric, undirected graph)

$$A = U \Sigma U^T$$



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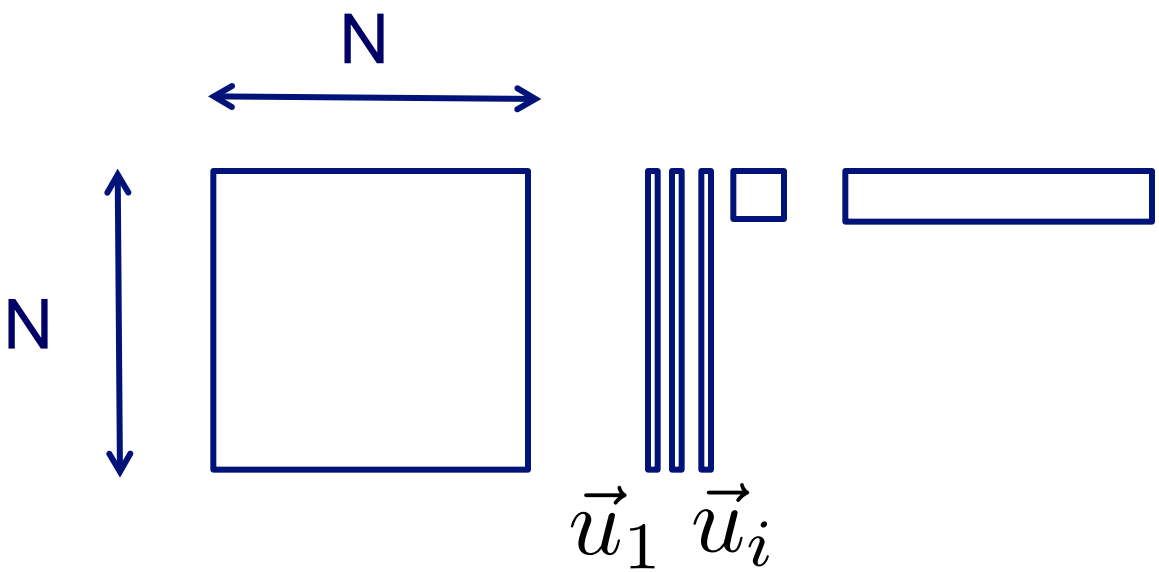

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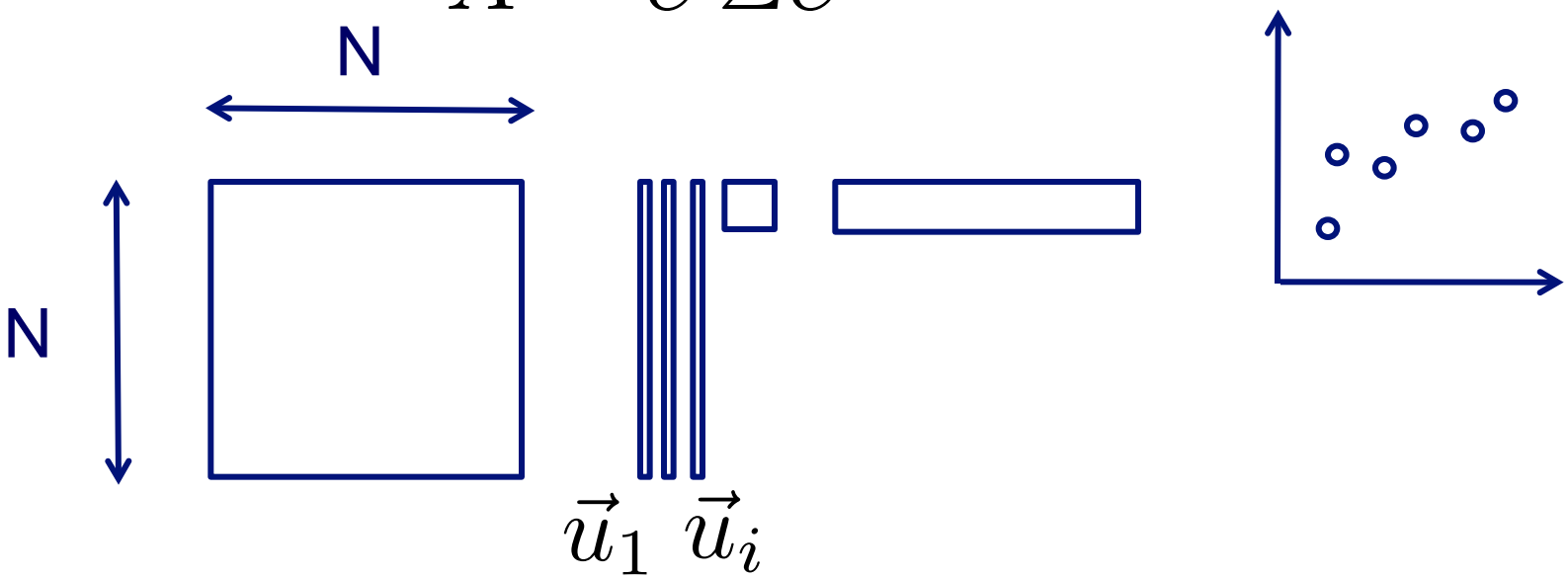

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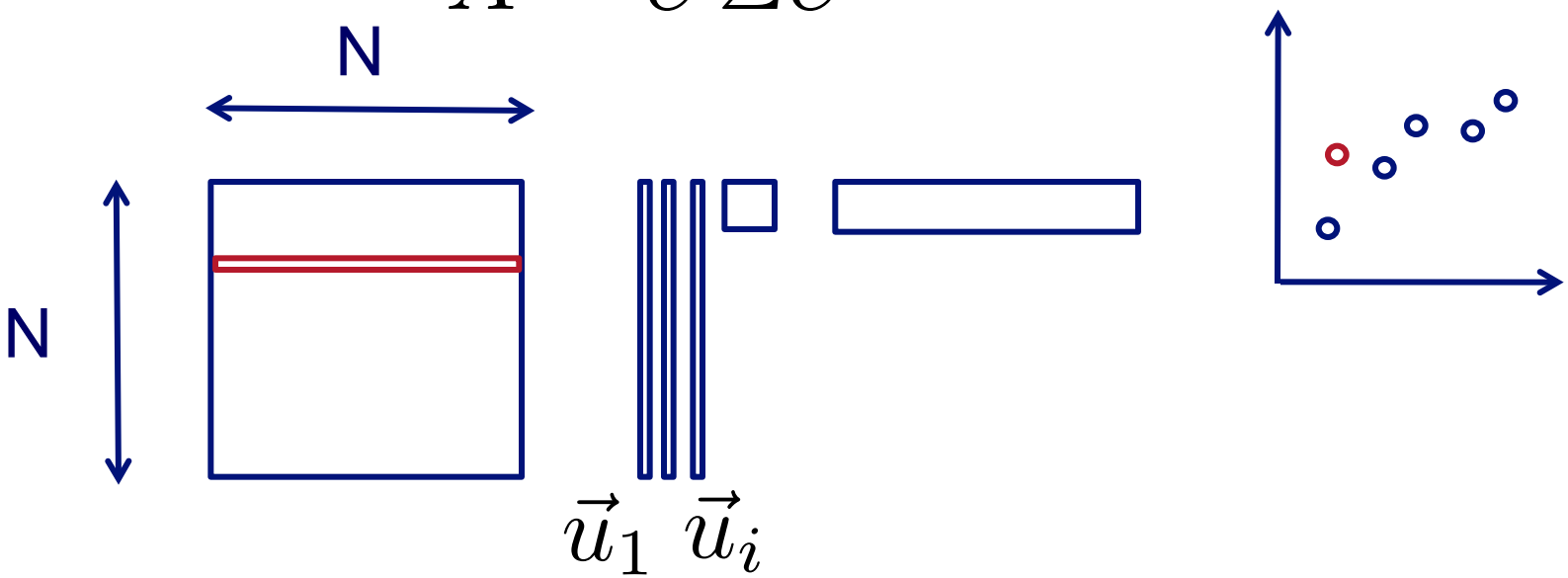

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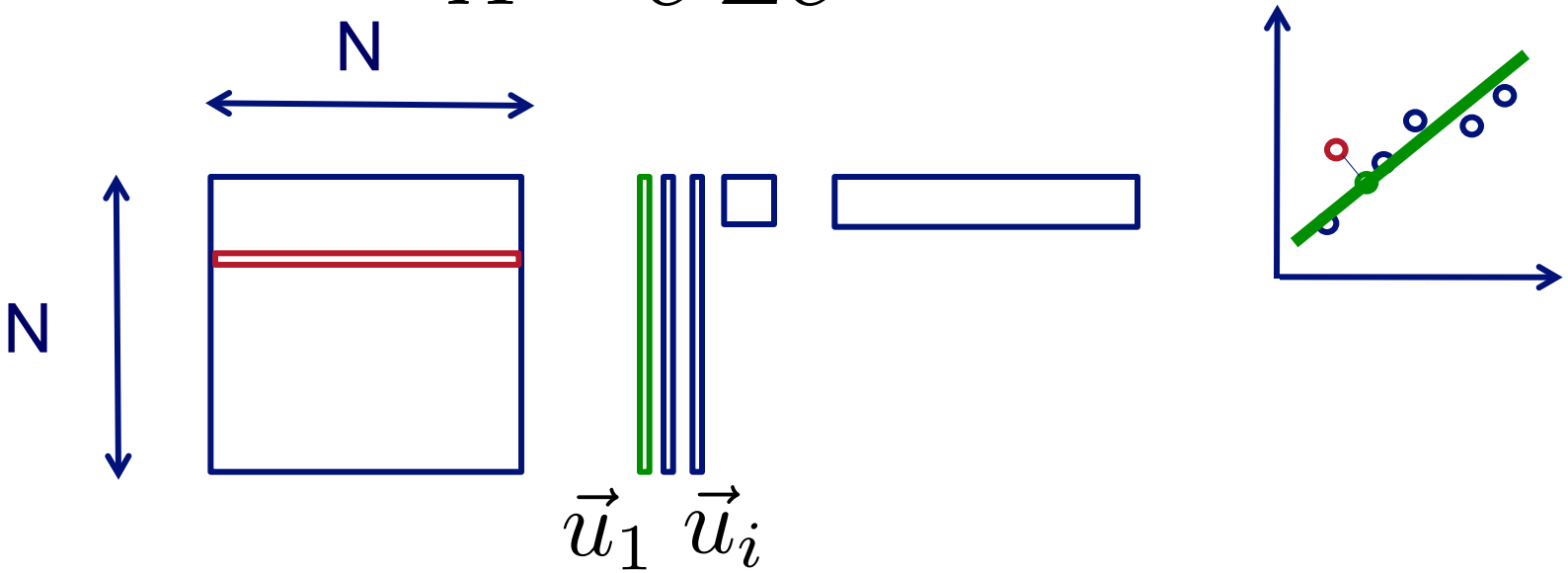
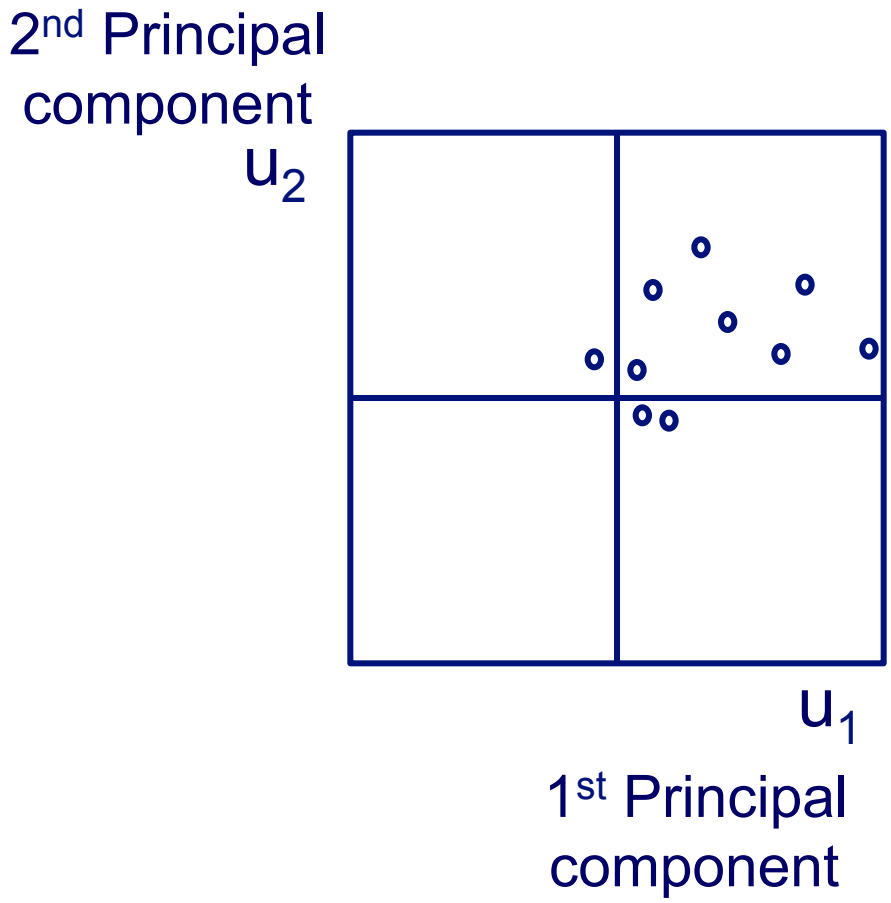


EigenSpokes

- EigenEigen (EE) Plot
  - Scatter plot of scores of u_1 vs. u_2
- One would expect
  - Many points @ origin
  - A few scattered ~randomly



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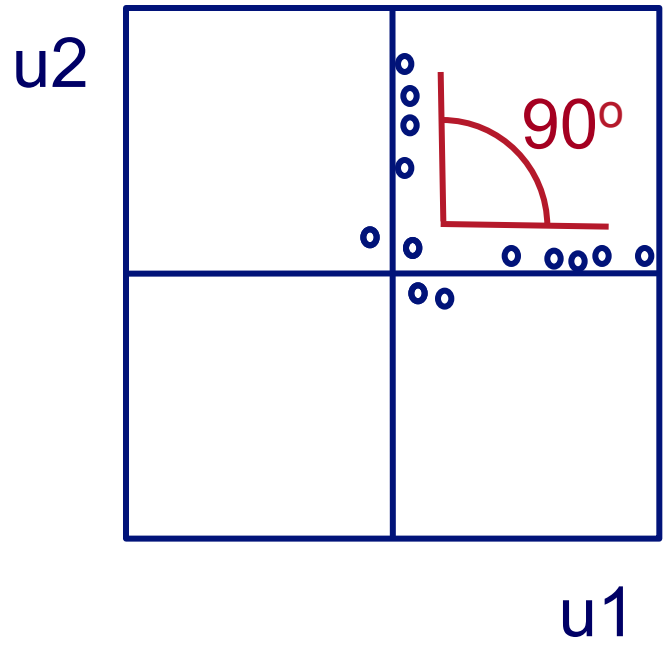
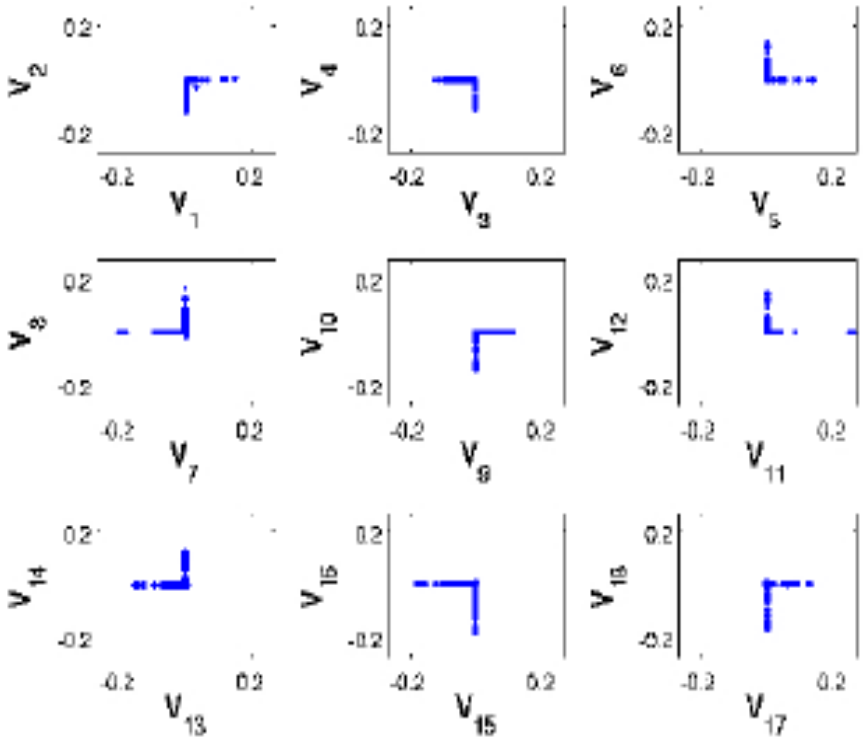


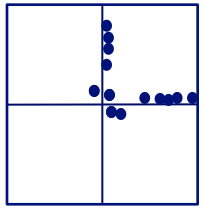
EigenSpokes - pervasiveness

- Present in mobile social graph
  - Across time and space

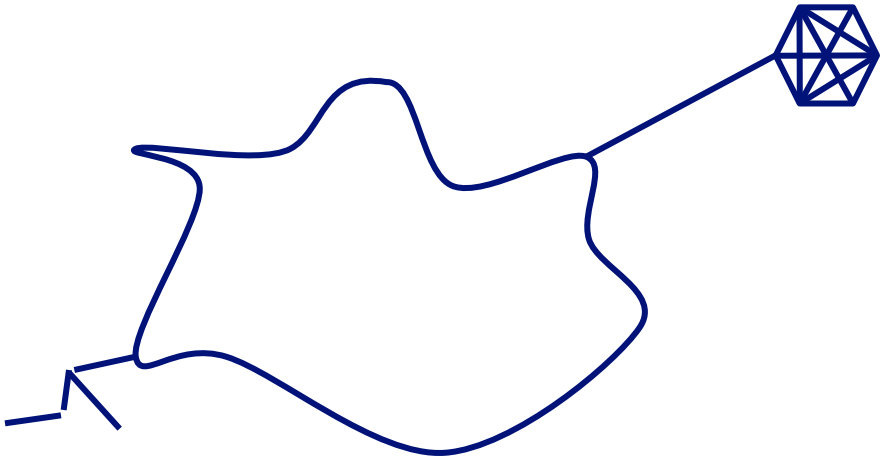
- Patent citation graph



EigenSpokes - explanation



Near-cliques, or near-bipartite-cores, loosely connected



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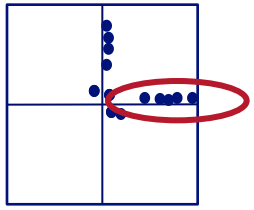
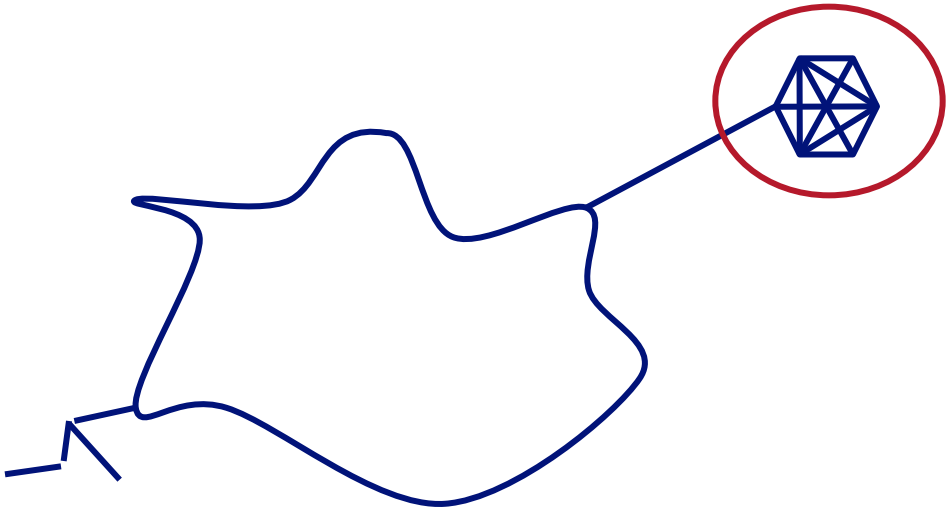
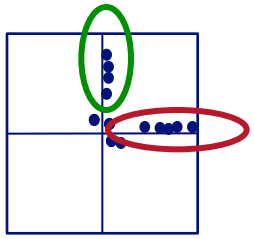


Near-cliques, or near-bipartite-cores, loosely connected



EigenSpokes - explanation



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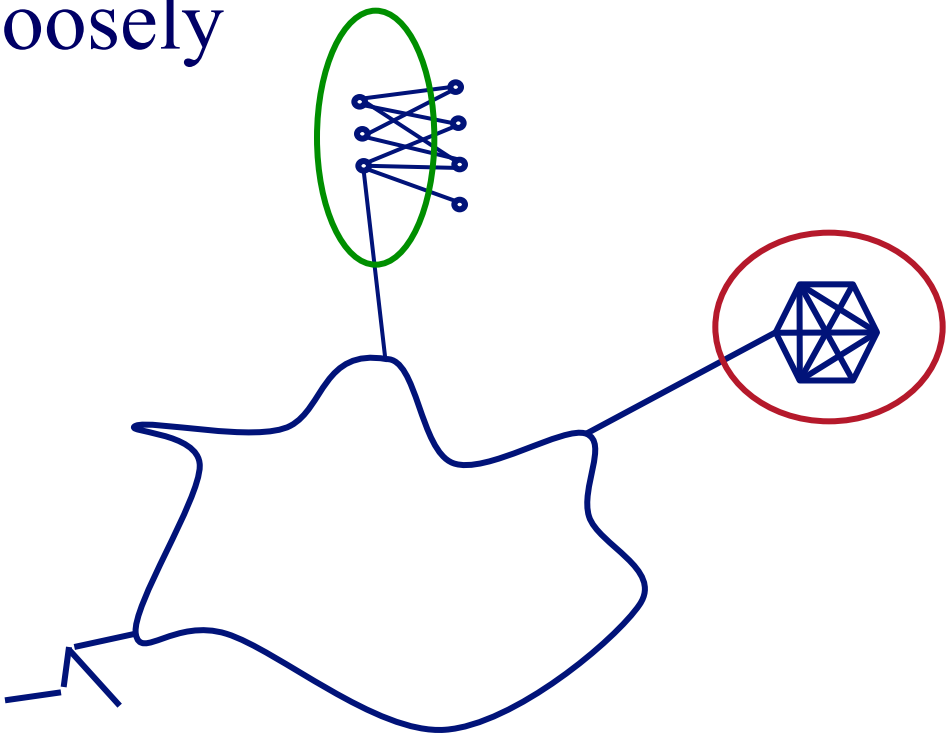
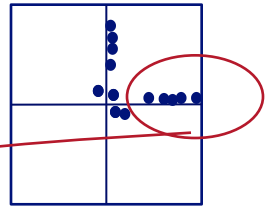
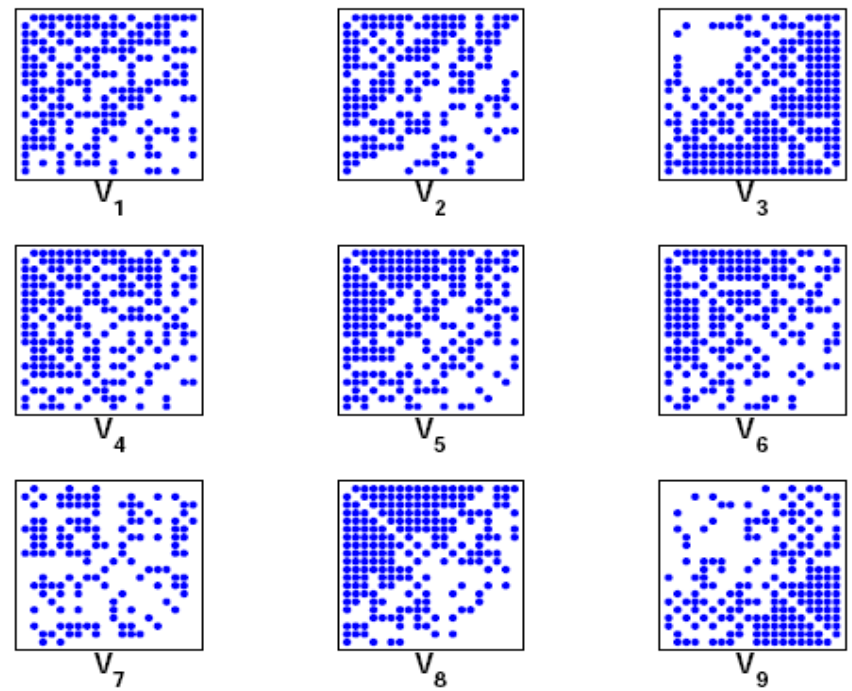


EigenSpokes - explanation

Near-cliques, or near-bipartite-cores, loosely connected



spy plot of top 20 nodes



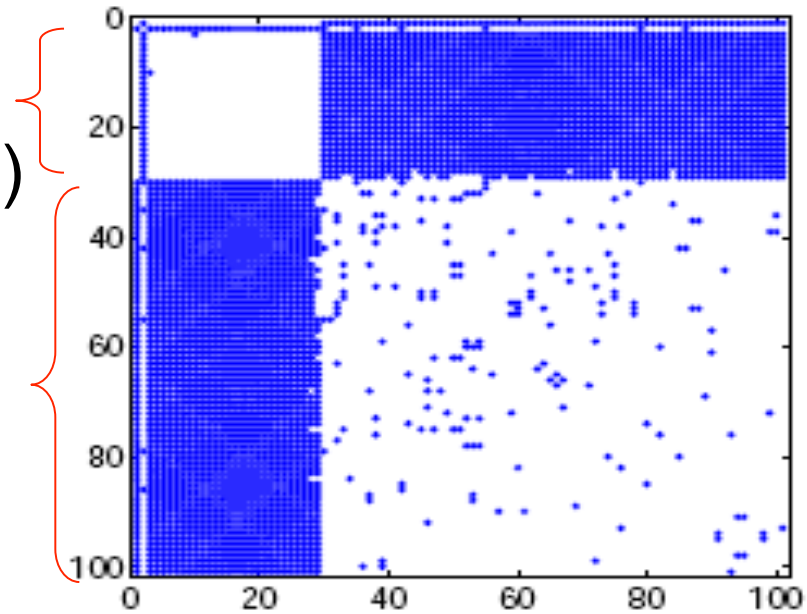
So what?

- Extract nodes with high s*cores*
- High connectivity
- Good "communities"

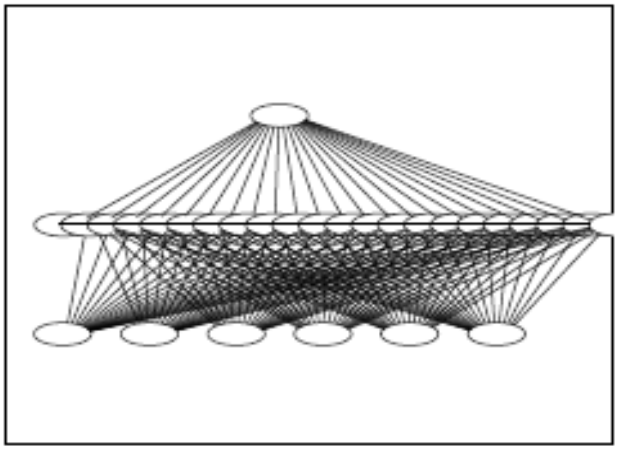
Bipartite Communities!

patents from
same inventor(s)

`cut-and-paste'
bibliography!



magnified bipartite community



Roadmap

- Patterns in graphs
  - Overview
  - Static graphs
  - Weighted graphs
  - Time-evolving graphs
- Anomaly Detection
- Application: ebay fraud
- Conclusions

Observations on Weighted Graphs

- A: yes - even more ‘laws’ !

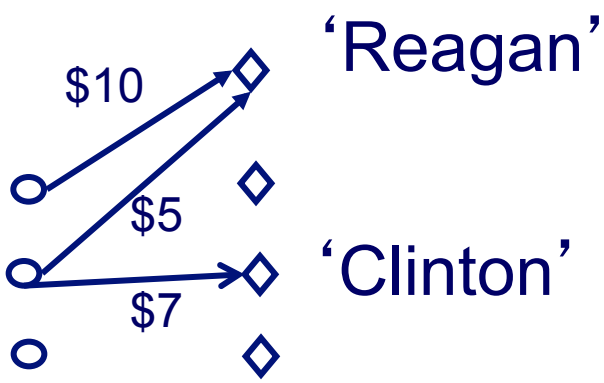
M. McGlohon, L. Akoglu, and C. Faloutsos
Weighted Graphs and Disconnected Components: Patterns and a Generator. KDD 2008

Observation W.1: Fortification

Q: How do the weights of nodes relate to degree?

Observation W.1: Fortification

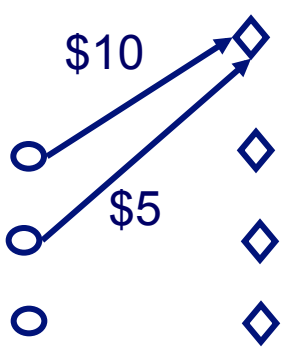
More donors, more $?



Observation W.1: fortification: Snapshot Power Law

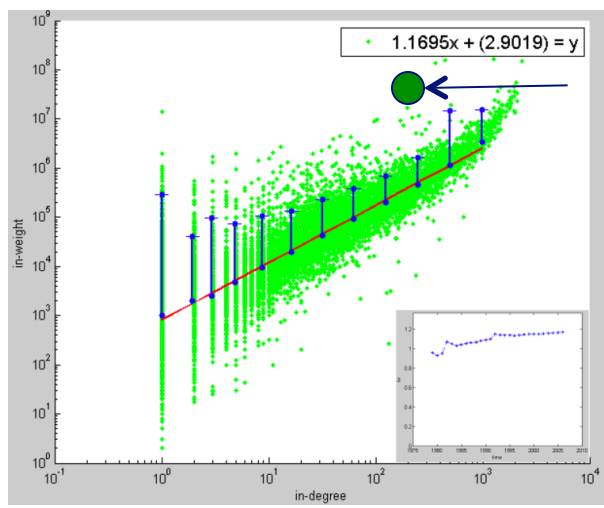
- Weight: super-linear on in-degree
- exponent 'iw': 1.01 < iw < 1.26

More donors, even more $



In-weights ($)

Orgs-Candidates



e.g. John Kerry, $10M received, from 1K donors

Edges (# donors)

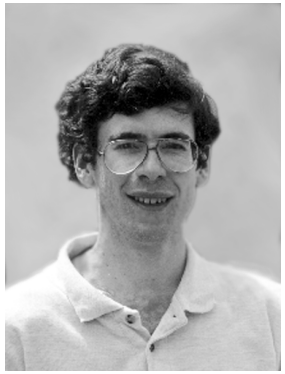
Roadmap

- Patterns in graphs
  - Overview
  - Static graphs
  - Weighted graphs
  - Time-evolving graphs
- Anomaly Detection
- Application: ebay fraud
- Conclusions

Problem: Time evolution

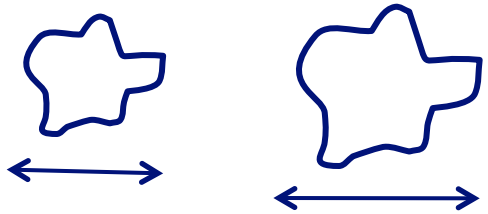
- with Jure Leskovec
  (CMU → Stanford)

- and Jon Kleinberg
  (Cornell – sabb. @ CMU)



T.1 Evolution of the Diameter

- Prior work on Power Law graphs hinted at **slowly growing diameter**:
  - diameter ~ O(log N)
  - diameter ~ O(log log N)



- What is happening in real data?

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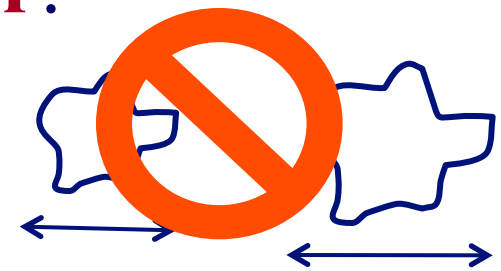
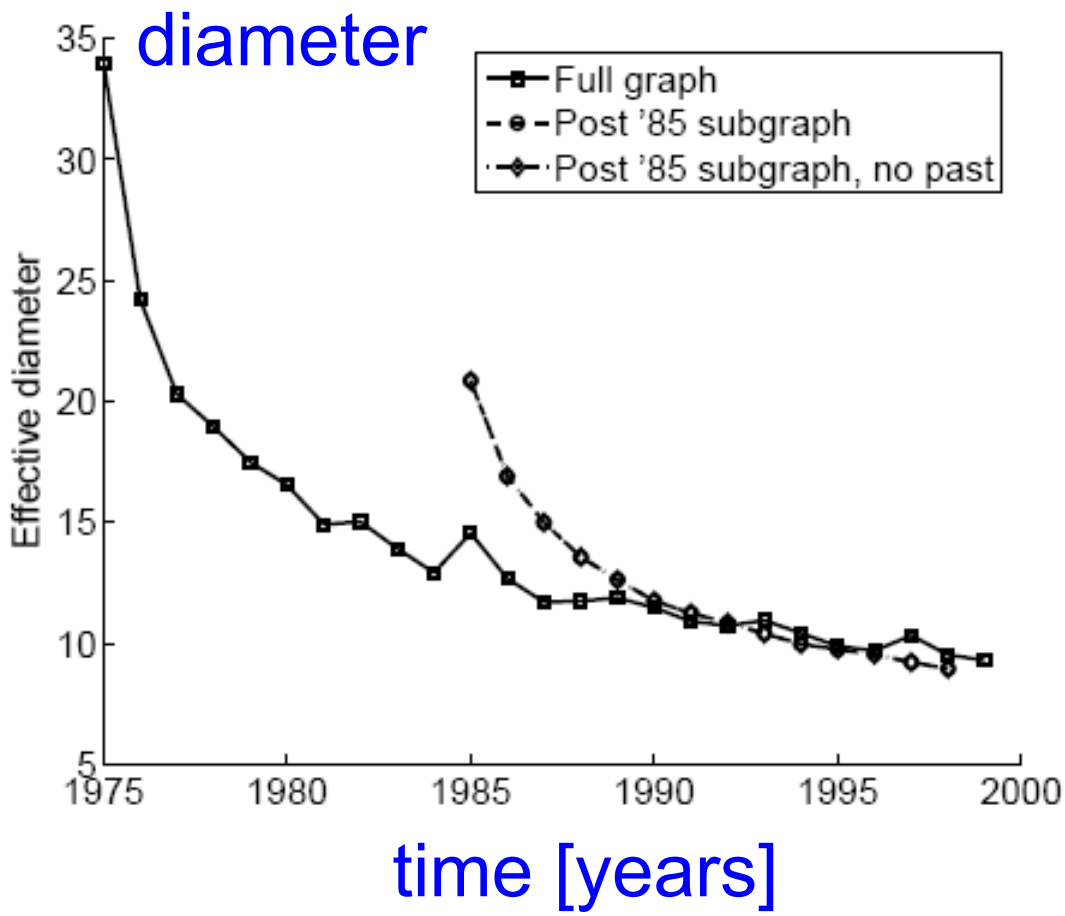


- What is happening in real data?
- Diameter **shrinks** over time

T.1 Diameter – "Patents"

- Patent citation network
- 25 years of data
- @1999
  - 2.9 M nodes
  - 16.5 M edges



T.2 Temporal Evolution of the Graphs

- N(t) … nodes at time t
- E(t) … edges at time t
- Suppose that

  $$N(t+1) = 2 * N(t)$$

- Q: what is your guess for

  $$E(t+1) =? 2 * E(t)$$

T.2 Temporal Evolution of the Graphs

- N(t) … nodes at time t
- E(t) … edges at time t
- Suppose that

  $$N(t+1) = 2 * N(t)$$

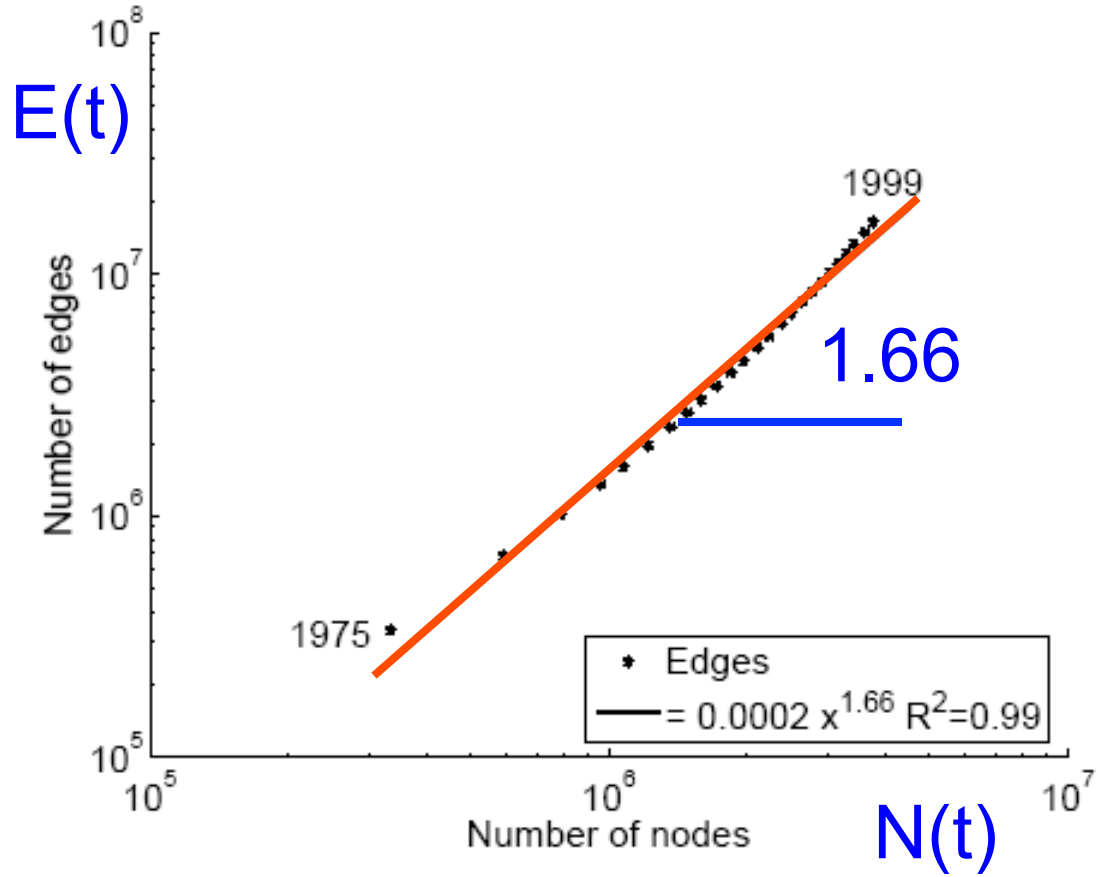
- Q: what is your guess for

  $$E(t+1) =? 2 * E(t)$$

- A: over-doubled!
  - But obeying the "Densification Power Law"

T.2 Densification – Patent Citations

- Citations among patents granted
- @1999
  - 2.9 M nodes
  - 16.5 M edges
- Each year is a datapoint



Roadmap

- Patterns in graphs
  - …
  - Time-evolving graphs
    - T1: Shrinking diameter
    - T2: Densification
    - T3: Connected components
    - T4: Popularity over time
    - T5: Phone-call patterns
- …

More on Time-evolving graphs

M. McGlohon, L. Akoglu, and C. Faloutsos. *Weighted Graphs and Disconnected Components: Patterns and a Generator.* KDD 2008

Observation T.3: NLCC behavior

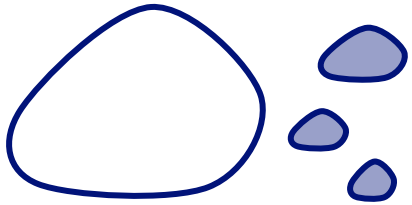
Q: How do the NLCCs emerge and join with the GCC?

("NLCC" = non-largest conn. components)

– Do they continue to grow in size?

– or do they shrink?

– or stabilize?



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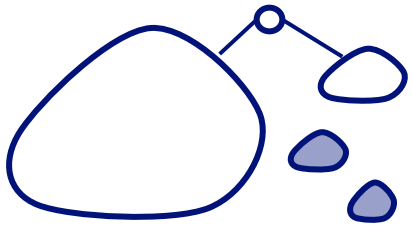


Observation T.3: NLCC behavior

Q: How do NLCCs emerge and join with the GCC?

("NLCC" = non-largest conn. components)

YES – Do they continue to grow in size?

YES – or do they shrink?

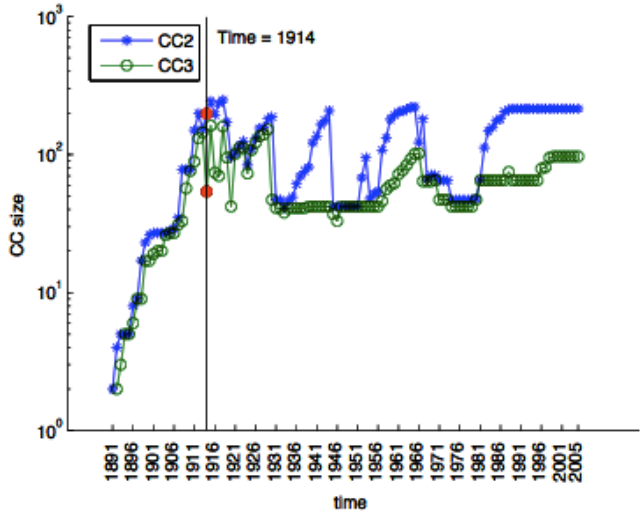
YES – or stabilize?

Observation T.3: NLCC behavior

- After the gelling point, the GCC takes off, but the NLCCs remain ~constant (actually, **oscillate**).

IMDB

CC size



Time-stamp

Roadmap

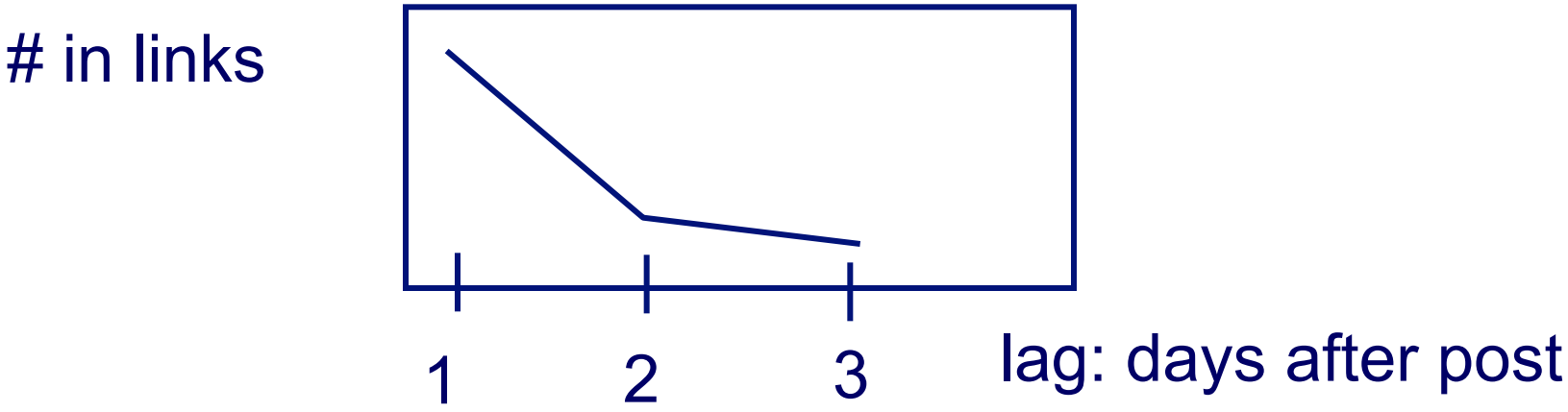
- Patterns in graphs
  - …
  - Time-evolving graphs
    - T1: Shrinking diameter
    - T2: Densification
    - T3: Connected components
    - T4: Popularity over time
    - T5: Phone-call patterns
- …

Timing for Blogs

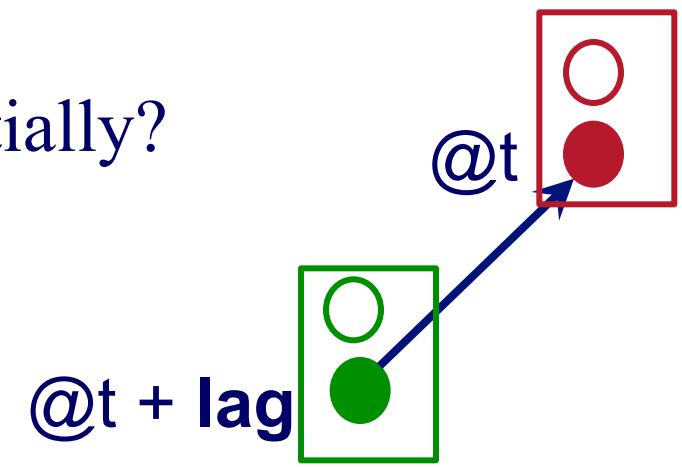
- with Mary McGlohon (CMU→Google)
- Jure Leskovec (CMU→Stanford)
- Natalie Glance (now at Google)
- Matt Hurst (now at MSR)

[SDM' 07]

T.4 : popularity over time

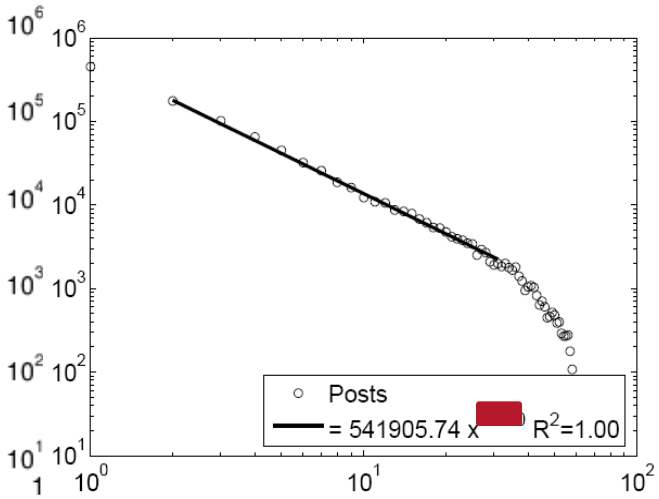


Post popularity drops-off – exponentially?



T.4 : popularity over time

in links
(**log**)

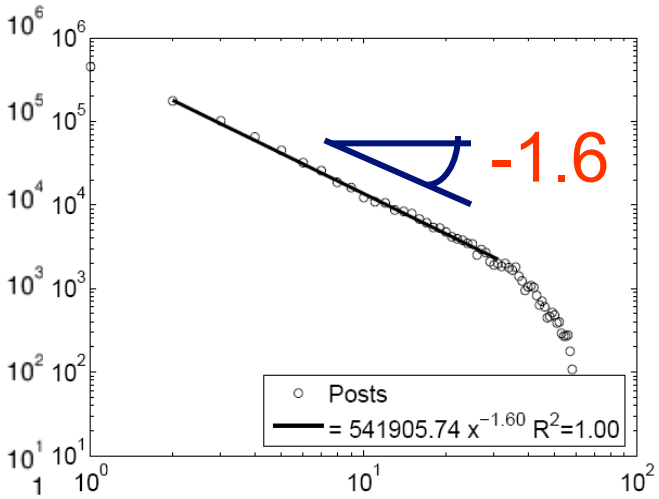


days after post
(**log**)

Post popularity drops-off – exponentially?
POWER LAW!
Exponent?

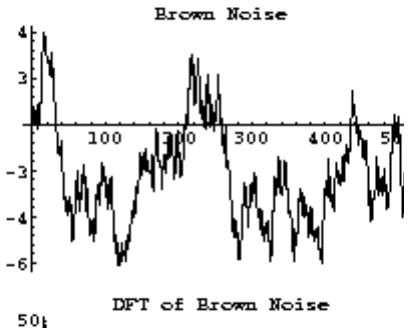
T.4 : popularity over time

in links
(**log**)



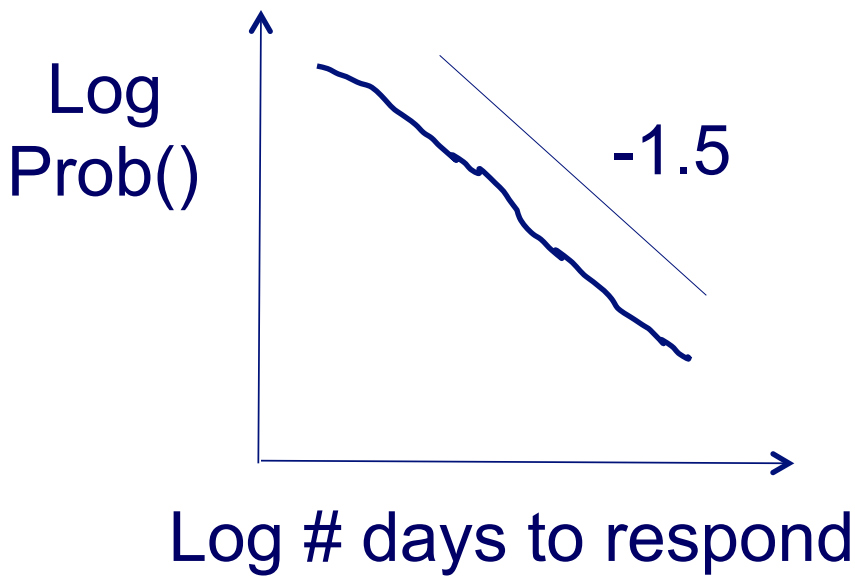
days after post
(**log**)

Post popularity drops-off – exponentially?
POWER LAW!
Exponent? -1.6
- close to -1.5: Barabasi's stack model
- and like the zero-crossings of a random walk



-1.5 slope

J. G. Oliveira & A.-L. Barabási. Human Dynamics: The Correspondence Patterns of Darwin and Einstein. *Nature* **437,** 1251 (2005) . [PDF]



Roadmap

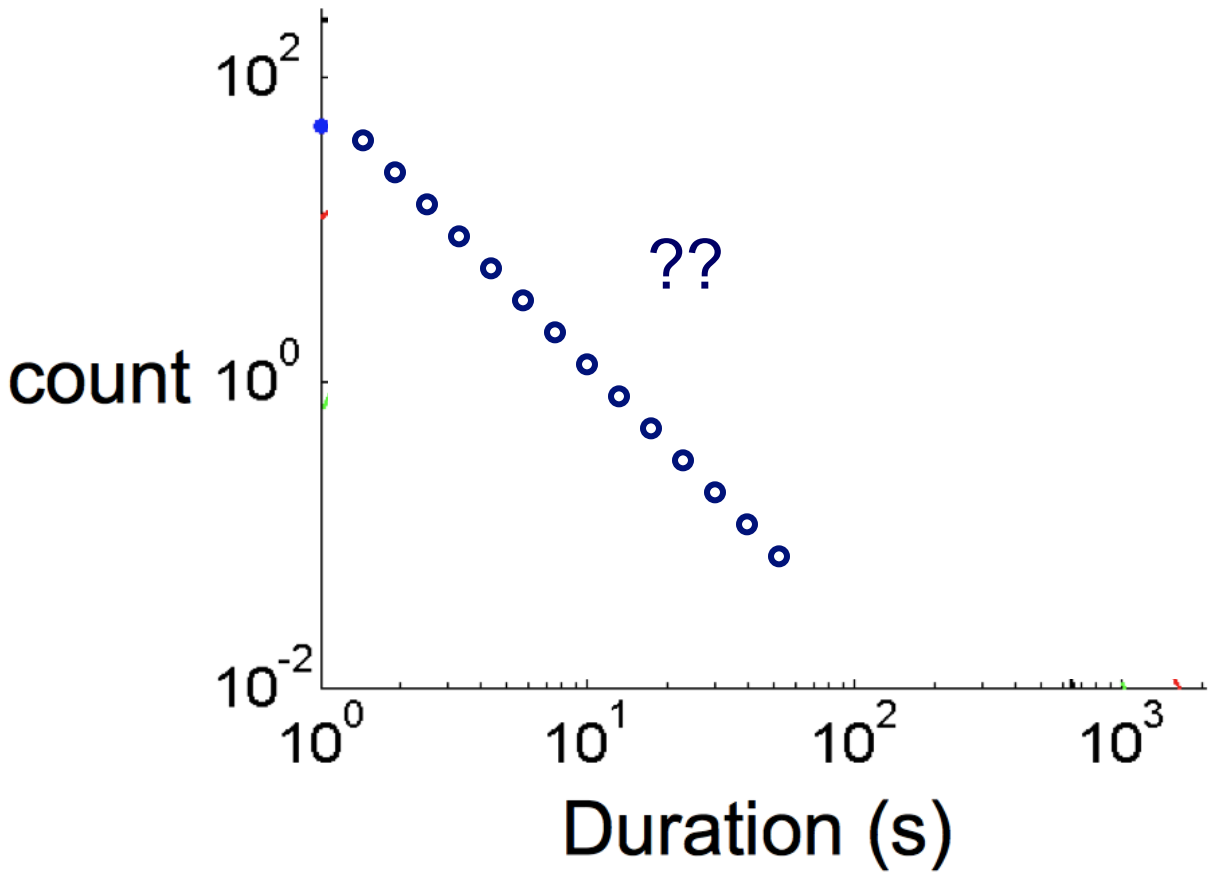
- Patterns in graphs
  - …
  - Time-evolving graphs
    - T1: Shrinking diameter
    - T2: Densification
    - T3: Connected components
    - T4: Popularity over time
    - T5: Phone-call patterns
- …

T.5: duration of phonecalls

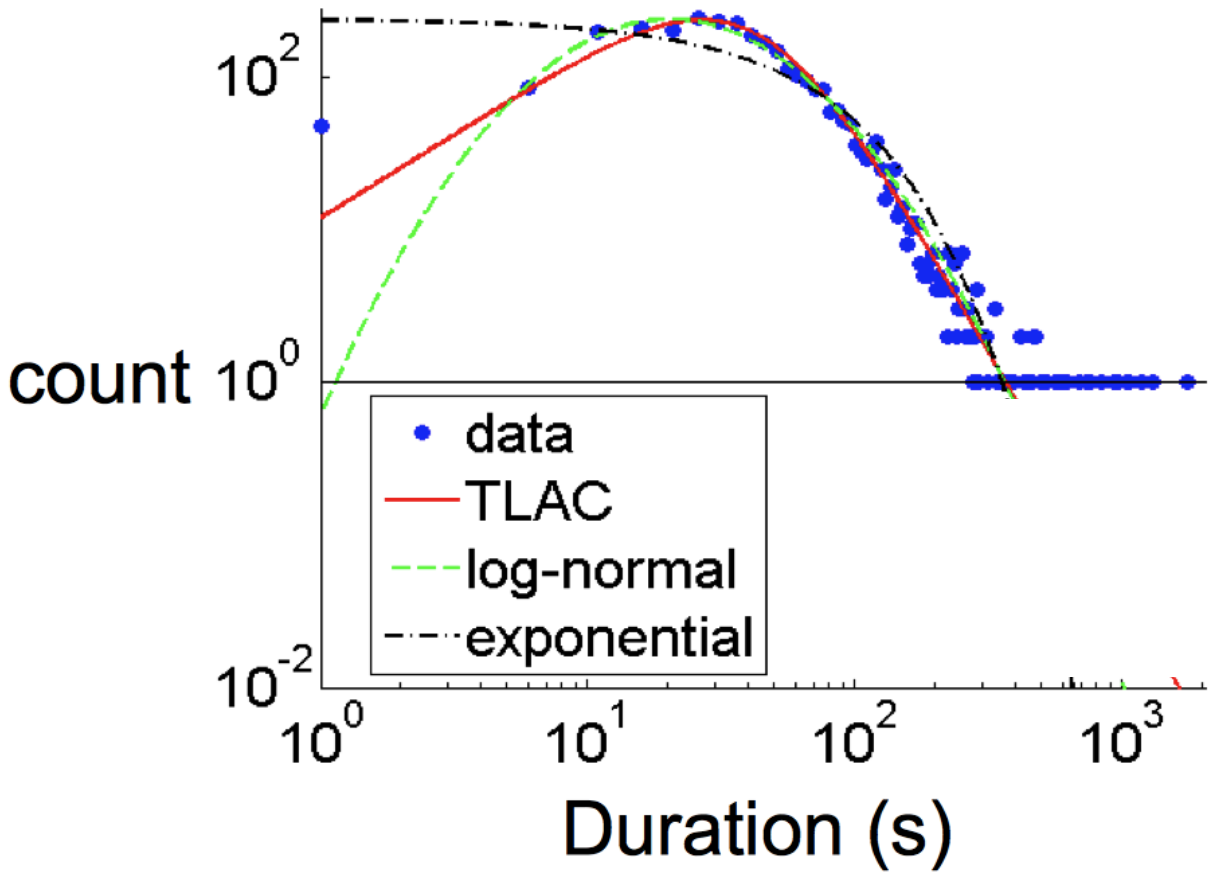
Surprising Patterns for the Call Duration Distribution of Mobile Phone Users.

Pedro O. S. Vaz de Melo, Leman Akoglu, Christos Faloutsos, Antonio A. F. Loureiro. PKDD 2010

Probably, power law (?)

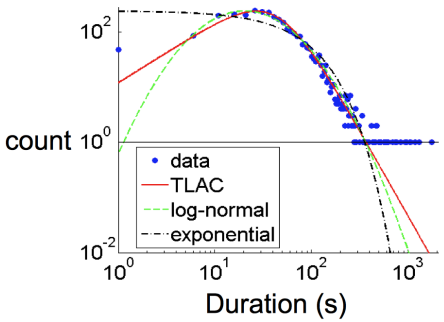


No Power Law!


count
10^2
10^0
10^{-2}
10^0
10^1
10^2
10^3
Duration (s)
data
TLAC
log-normal
exponential

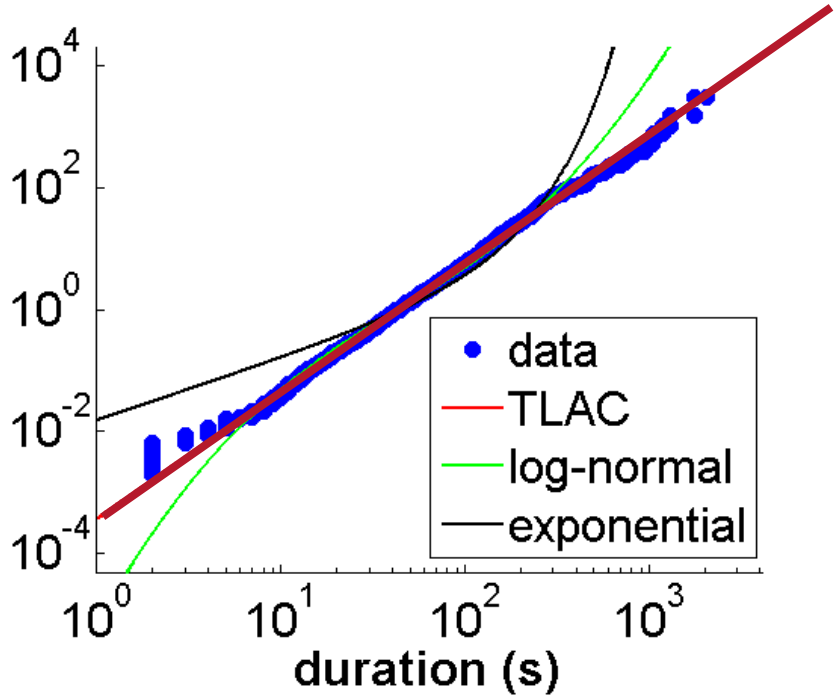
'TLaC: Lazy Contractor'

- The longer a task (phone-call) has taken, the even longer it will take



Odds ratio=
Casualties(<x):
Survivors(>=x)

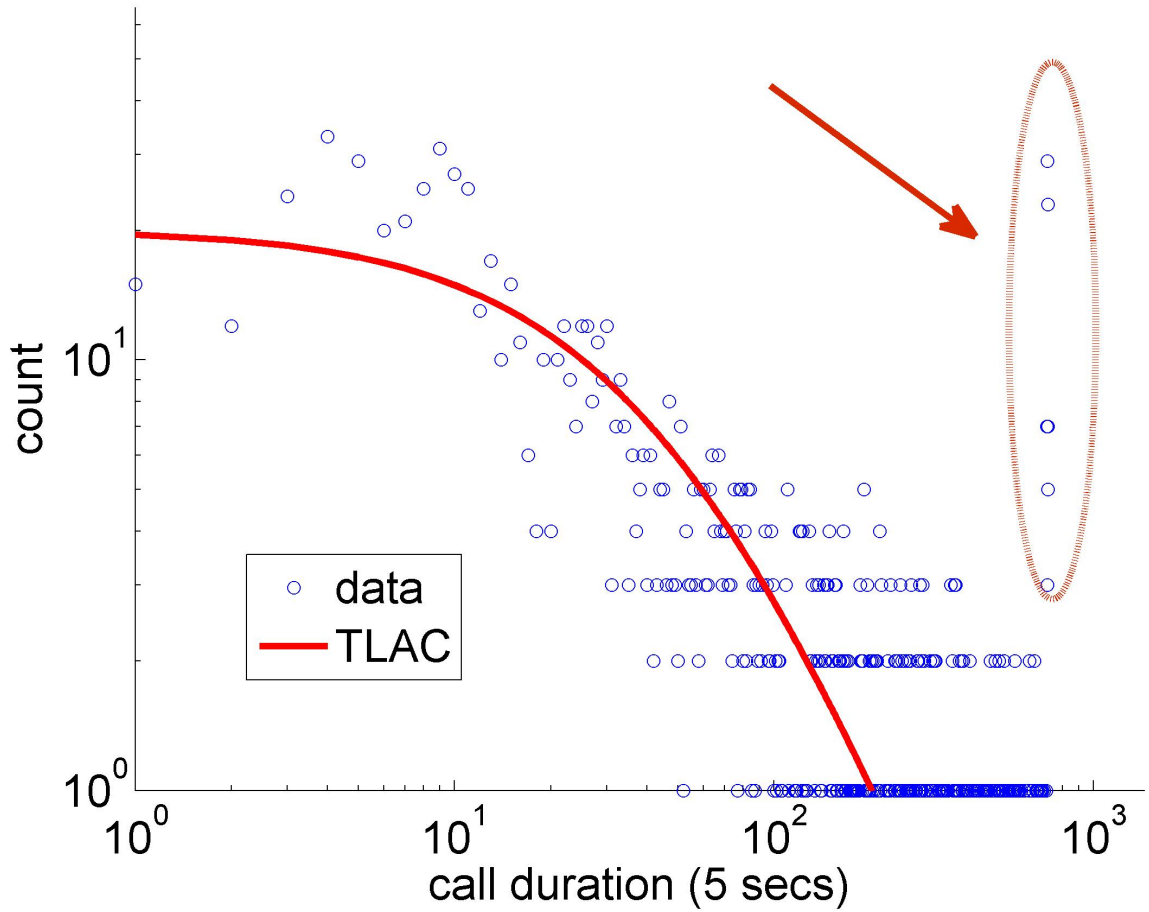
== power law



Data Description

- Data from a private mobile operator of a large city
  - 4 months of data
  - 3.1 million users
  - more than 1 billion phone records
- Over 96% of ‘talkative’ users obeyed a TLAC distribution (‘talkative’: >30 calls)

Outliers:



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Carnegie Mellon

Real Graph Patterns

	unweighted	weighted
static	✓ **P01**. Power-law degree distribution [Faloutsos et. al.`99, Kleinberg et. al.`99, Chakrabarti et. al. `04, Newman`04] ✓ **P02**. Triangle Power Law [Tsourakakis `08] ✓ **P03**. Eigenvalue Power Law [Siganos et. al. `03] **P04**. Community structure [Flake et. al.`02, Girvan and Newman `02] ✓ **P05**. Clique Power Laws [Du et. al. '09]	**P12**. Snapshot Power Law [McGlohon et. al. `08]
dynamic	✓ **P06**. Densification Power Law [Leskovec et. al.`05] ✓ **P07**. Small and shrinking diameter [Albert and Barabási `99, Leskovec et. al. '05, McGlohon et. al. '08] **P08**. Gelling point [McGlohon et. al. `08] ✓ **P09**. Constant size 2nd and 3rd connected components [McGlohon et. al. `08] **P10**. Principal Eigenvalue Power Law [Akoglu et. al. `08] **P11**. Bursty/self-similar edge/weight additions [Gomez and Santonja `98, Gribble et. al. `98, Crovella and Bestavros `99, McGlohon et .al. `08]	✓ **P13**. Weight Power Law [McGlohon et. al. `08] ✓ **P14**. Skewed call duration distributions [Vaz de Melo et. al. `10]

RTG: A Recursive Realistic Graph Generator using Random Typing

Leman Akoglu and Christos Faloutsos. *ECML PKDD*'09.

Roadmap

- Patterns in graphs
  - Overview
  - Static graphs
  - Weighted graphs
  - Time-evolving graphs
- Anomaly Detection
- Application: ebay fraud
- Conclusions

OddBall: Spotting Anomalies in Weighted Graphs

Leman Akoglu, Mary McGlohon,
Christos Faloutsos

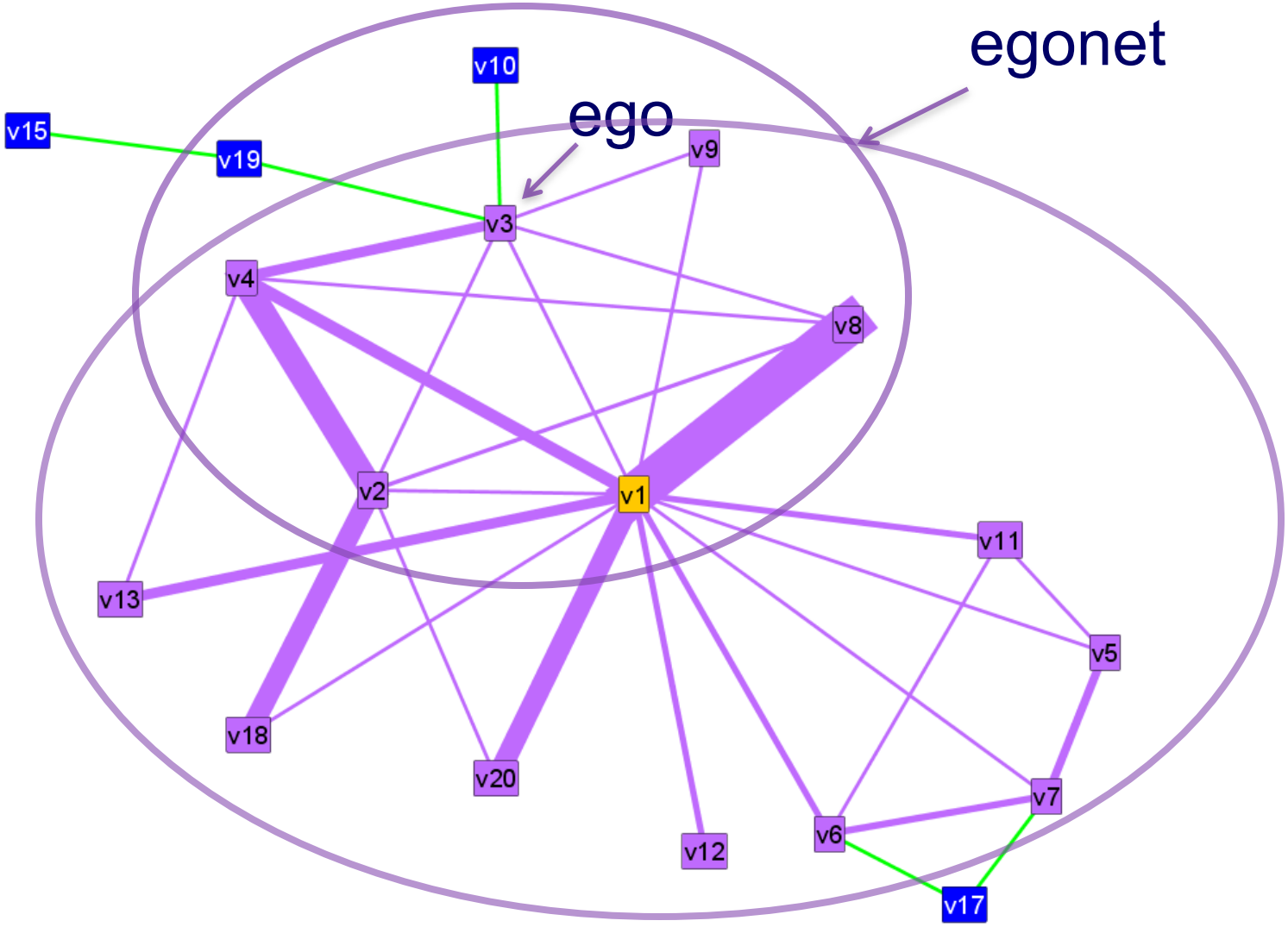
PAKDD 2010, Hyderabad, India

Main idea

For each node,

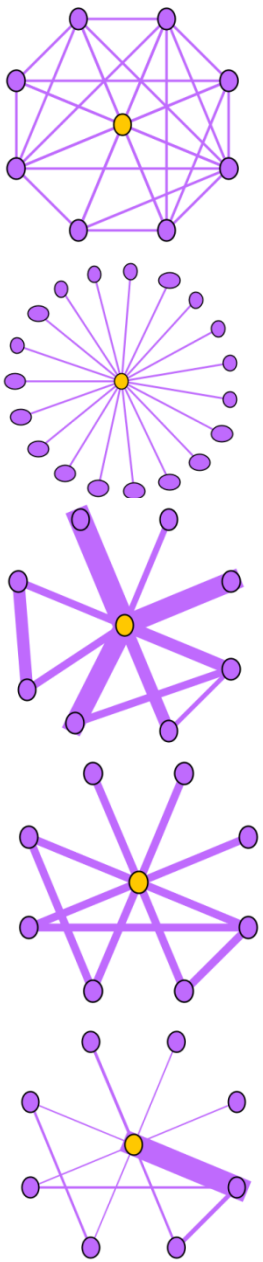
- Extract 'ego-net' (=1-step-away neighbors)
- Extract features (#edges, total weight, *etc*)
- Compare with the rest of the population

What is an egonet?

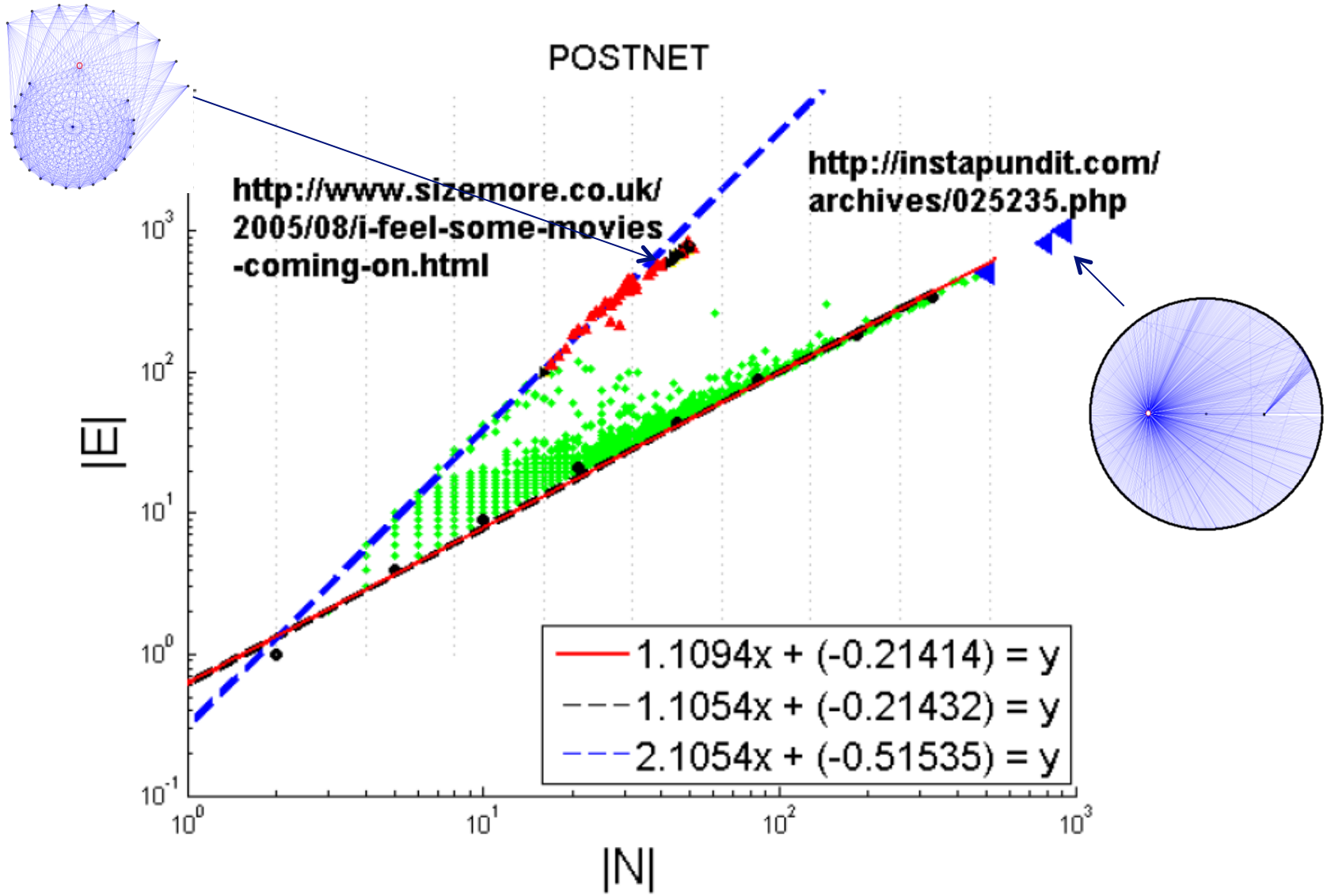


Selected Features

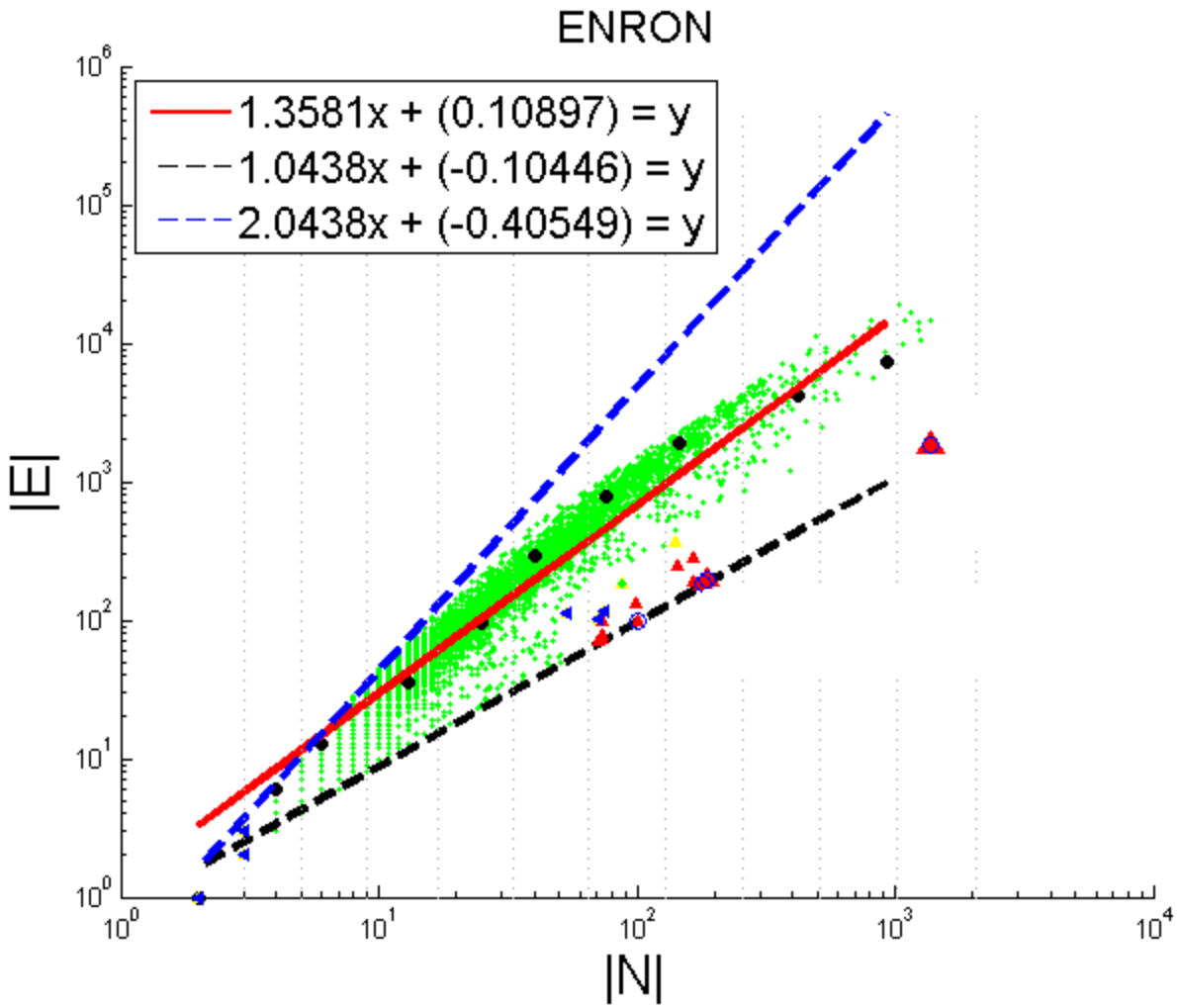
- N_i: number of neighbors (degree) of ego i
- E_i: number of edges in egonet i
- W_i: total weight of egonet i
- $\lambda_{w,i}$: principal eigenvalue of the weighted adjacency matrix of egonet I



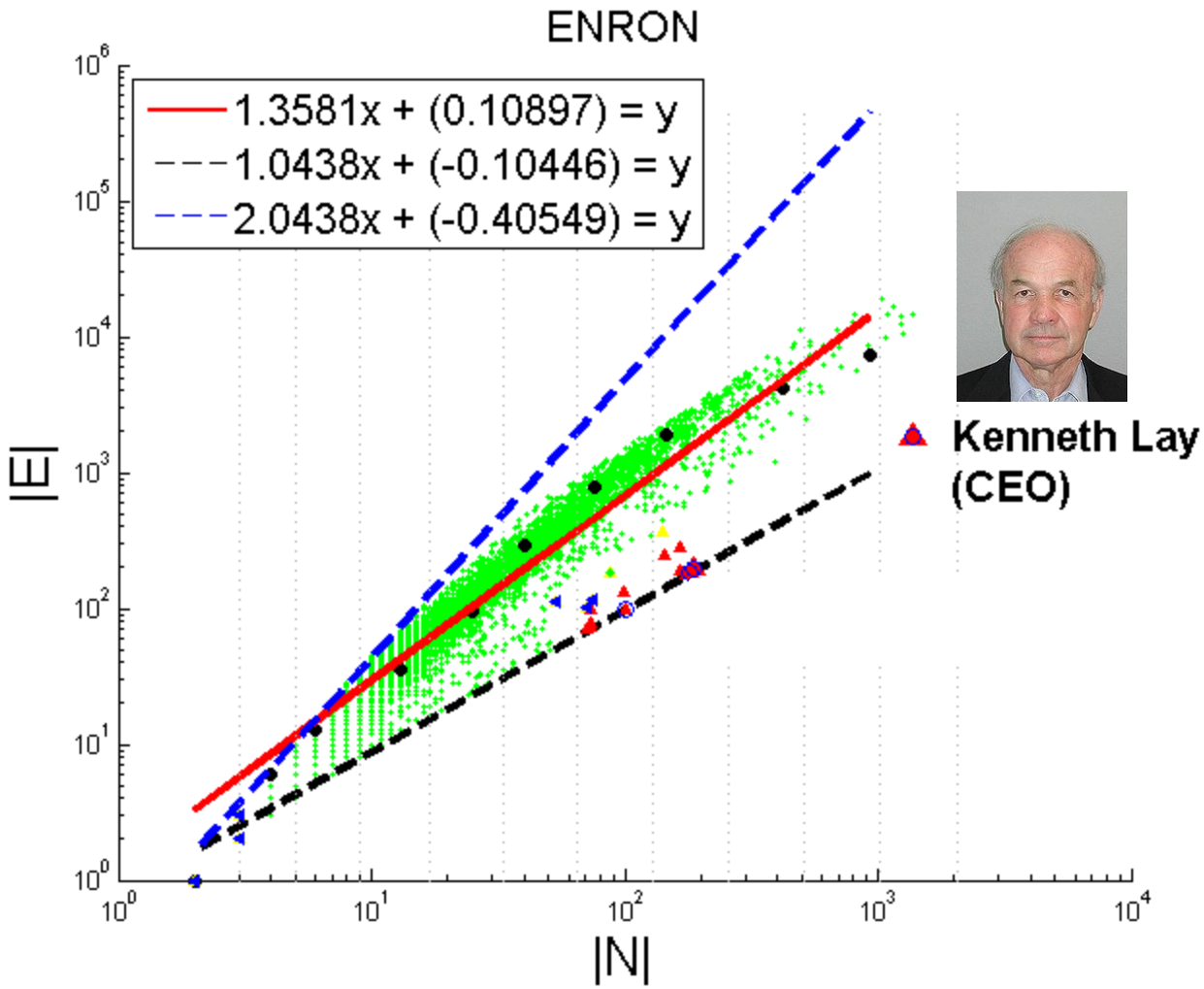
Near-Clique/Star



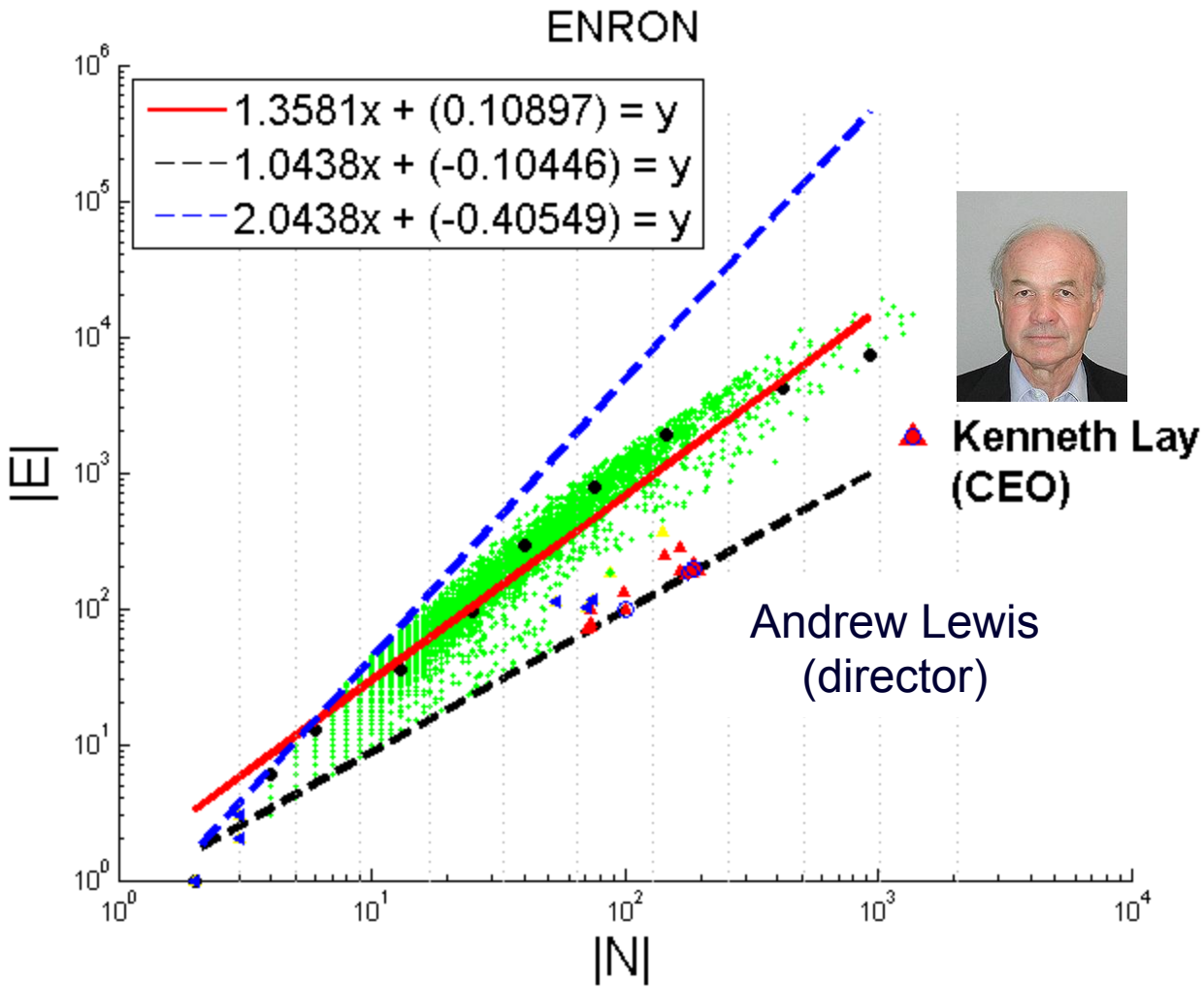
Near-Clique/Star



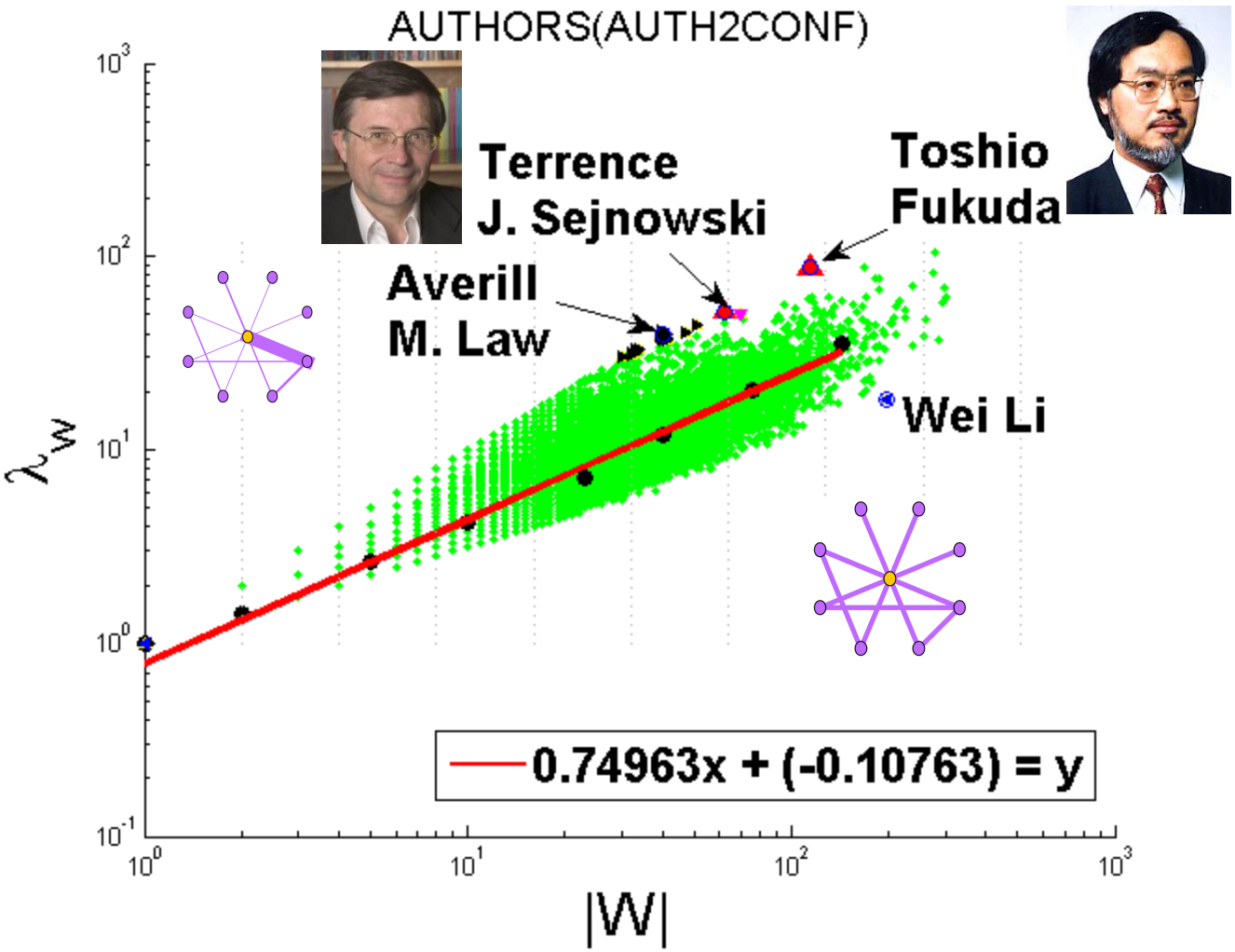
Near-Clique/Star



Near-Clique/Star



Dominant Heavy Link

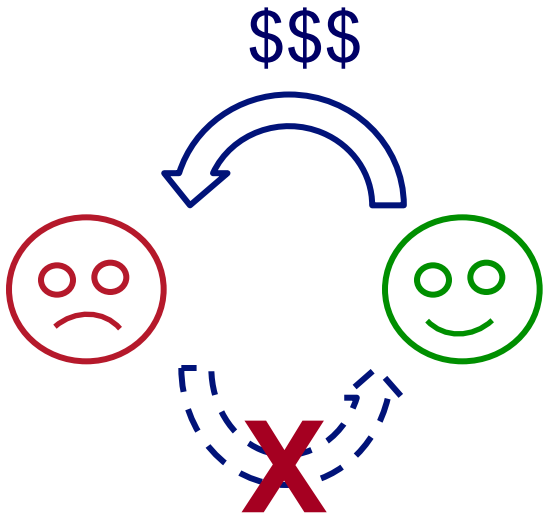


Roadmap

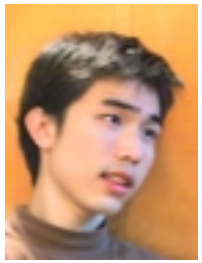
- Patterns in graphs
  - overview
  - Static graphs
  - Weighted graphs
  - Time-evolving graphs
- Anomaly Detection
- Application: ebay fraud
- Conclusions

NetProbe: The Problem

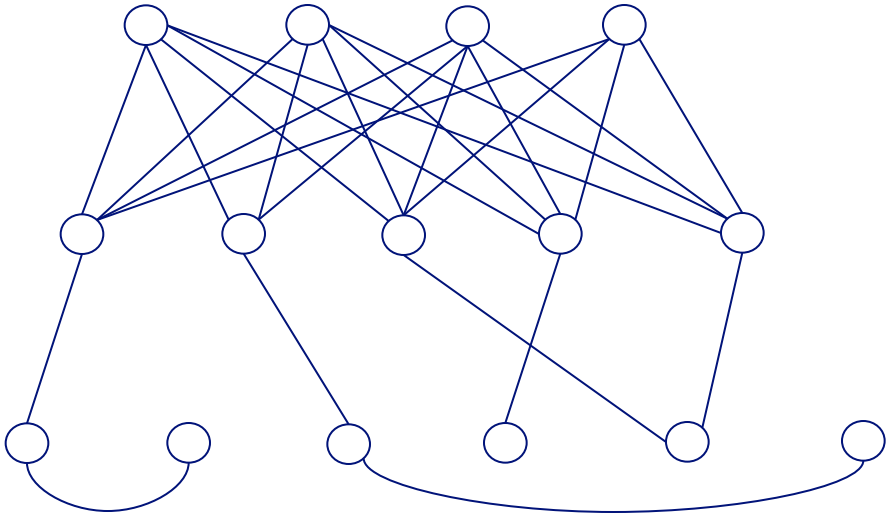
Find **bad sellers** (**fraudsters**) on eBay who don't deliver their (expensive) items



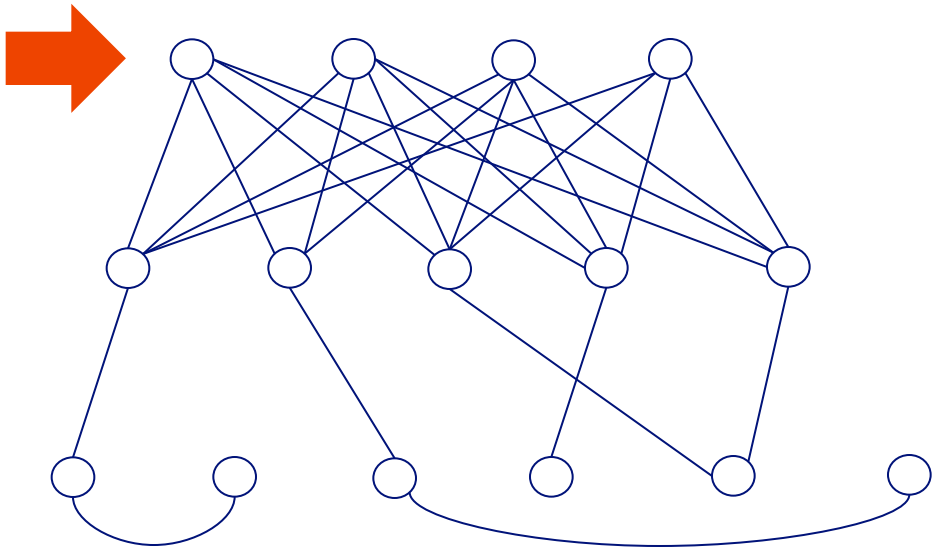
E-bay Fraud detection



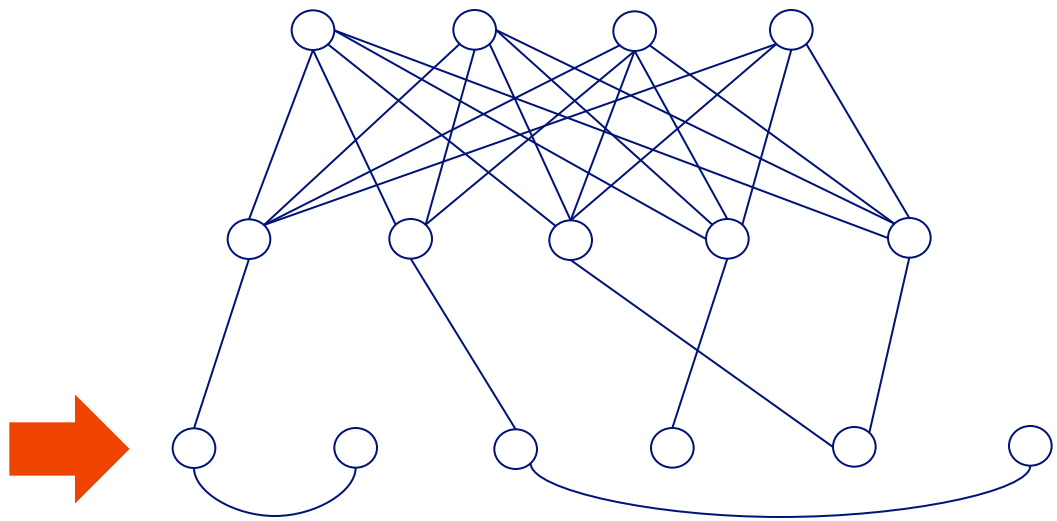
w/ Polo Chau &
Shashank Pandit, CMU
[www' 07]



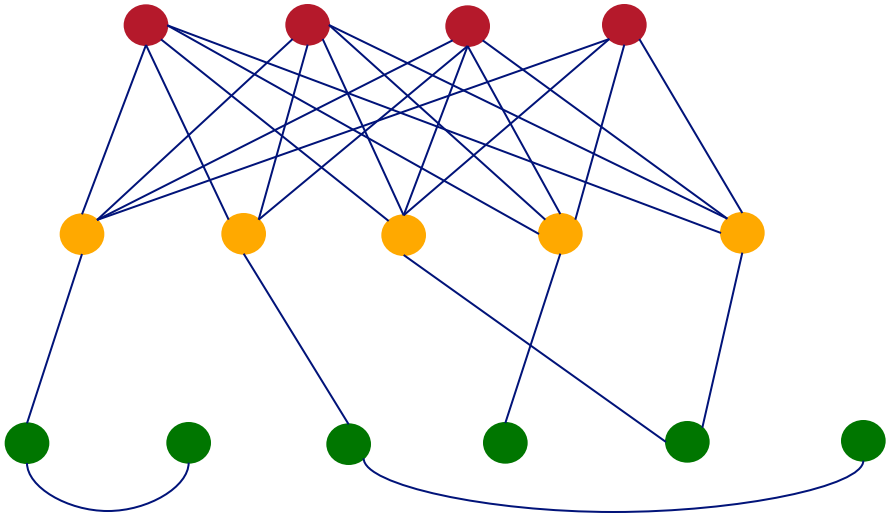
E-bay Fraud detection



E-bay Fraud detection

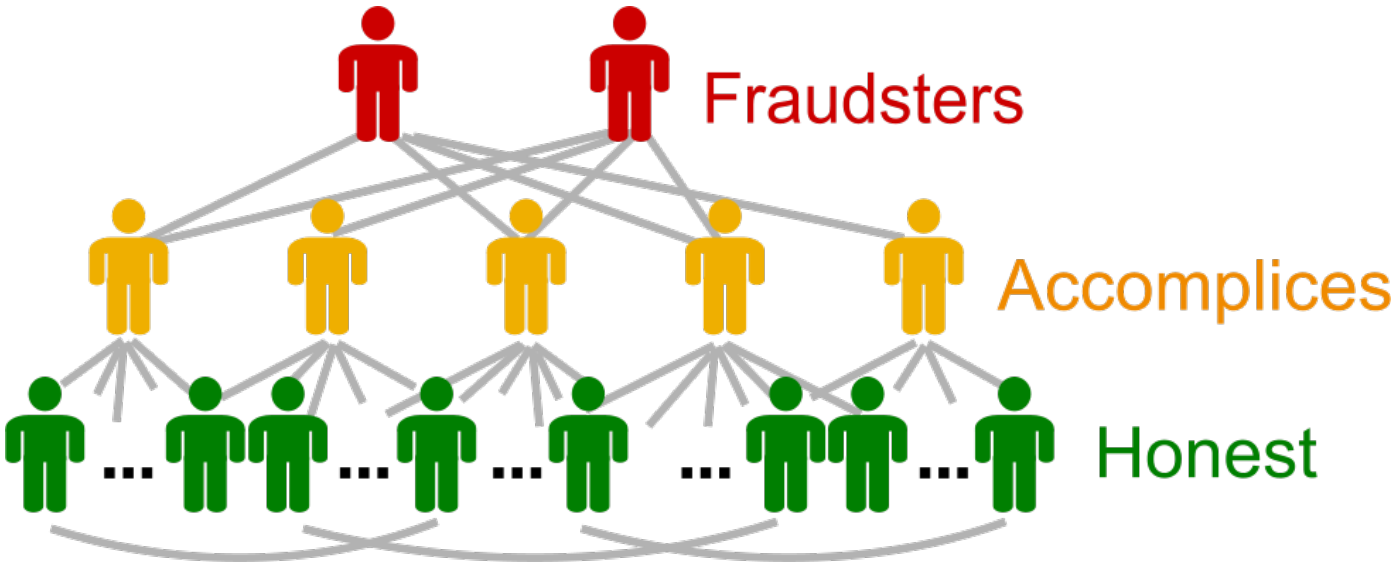


E-bay Fraud detection - NetProbe



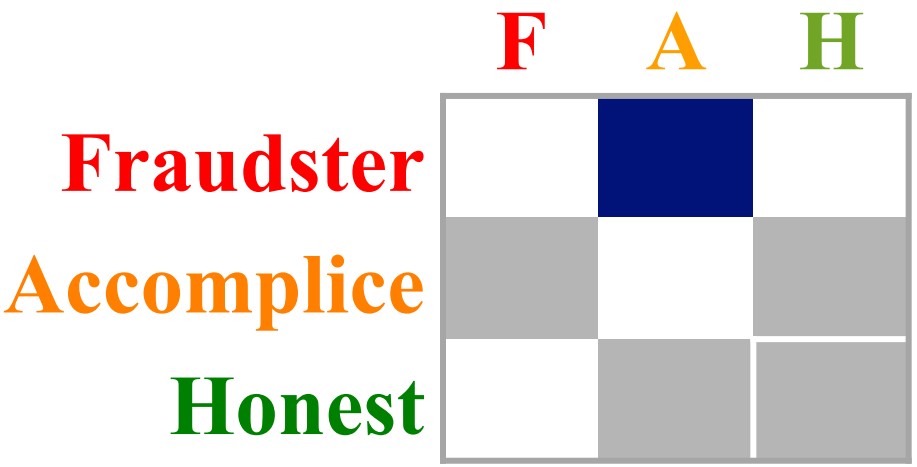
NetProbe: Key Ideas

- Fraudsters **fabricate their reputation** by "trading" with their accomplices
- Transactions form **near bipartite cores**
- How to detect them?

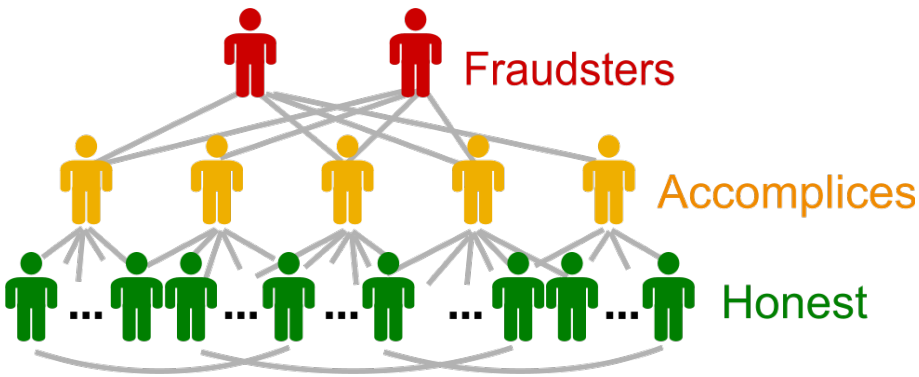


NetProbe: Key Ideas

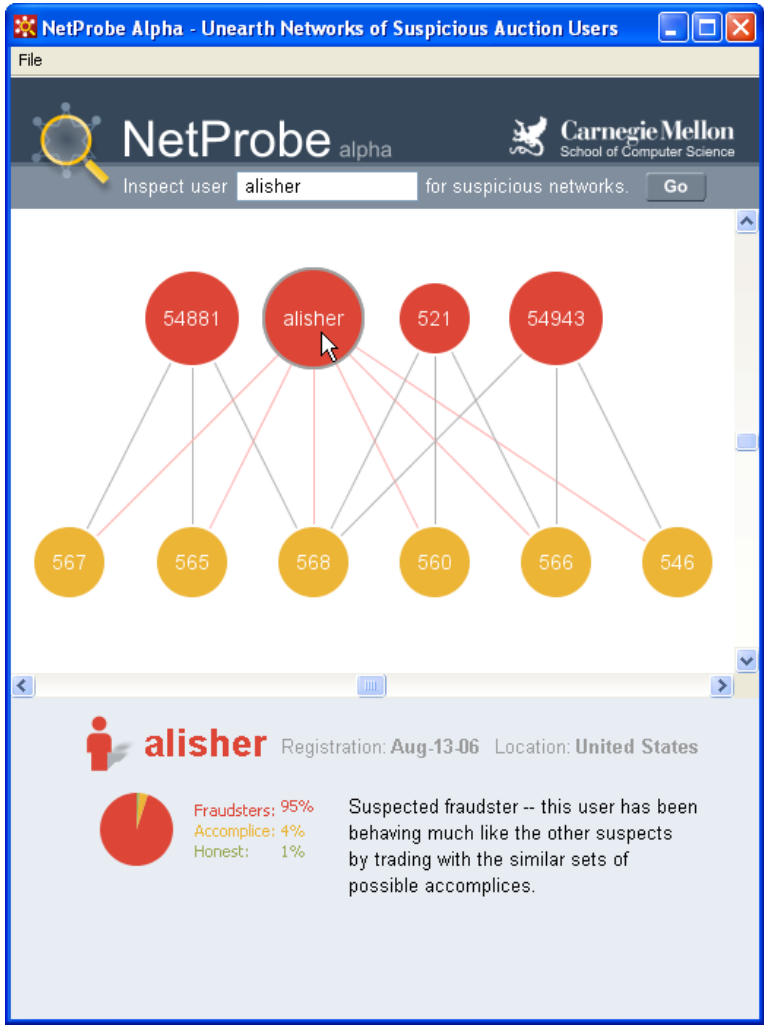
Use ‘**Belief Propagation**’ and ~**heterophily**

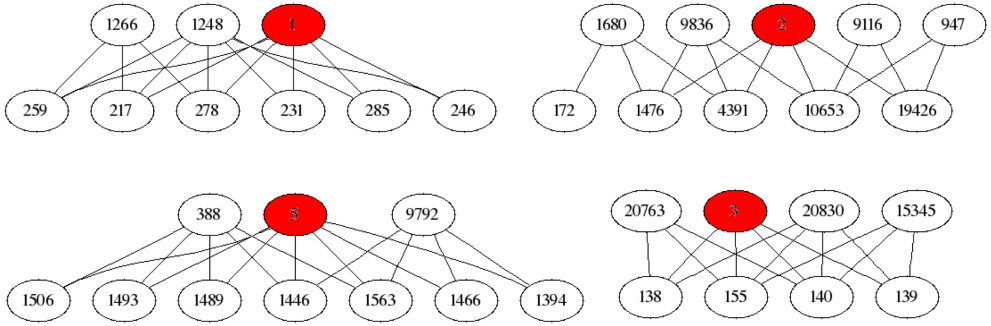


Darker means more likely



NetProbe: Main Results

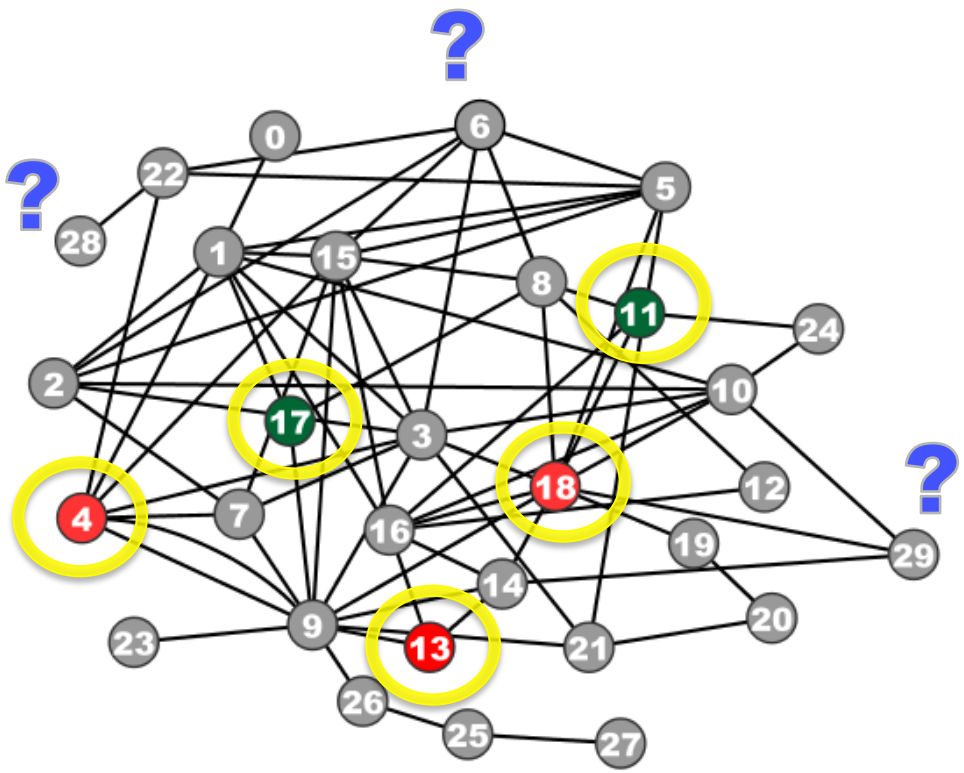




Roadmap

- Patterns in graphs
- Anomaly Detection
- Application: ebay fraud
  - How-to: Belief Propagation
- Conclusions

Guilt-by-Association Techniques

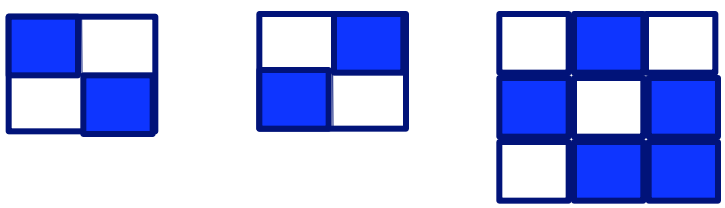


Given:
- graph and
- few labeled nodes

Find: class (red/green) for rest nodes

Assuming: network effects (homophily/ heterophily, etc)

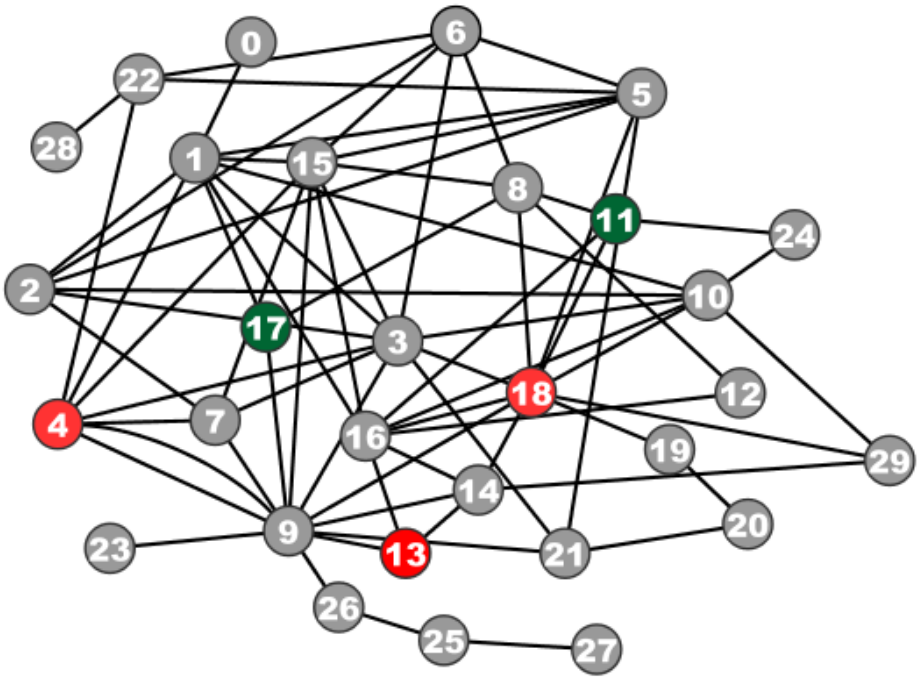
red
green



F
A
H

Correspondence of Methods

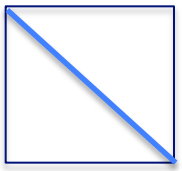
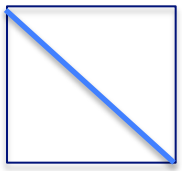
Random Walk with Restarts (**RWR**) Google
Semi-supervised Learning (**SSL**)
Belief Propagation (**BP**) Bayesian



Correspondence of Methods

Random Walk with Restarts (**RWR**) ≈
Semi-supervised Learning (**SSL**) ≈
Belief Propagation (**BP**)

Method	Matrix		unknown		known
RWR	$[\mathbf{I} - c\ \mathbf{AD}^{-1}]$	×	$\mathbf{x}$	=	$(1-c)\mathbf{y}$
SSL	$[\mathbf{I} + \alpha(\mathbf{D} - \mathbf{A})]$	×	$\mathbf{x}$	=	$\mathbf{y}$
FABP	$[\mathbf{I} + a\,\mathbf{D} - c'\ \mathbf{A}]$	×	$\mathbf{b}_h$	=	$\boldsymbol{\phi}_h$

$$\begin{bmatrix} 0 & 1 & 0 \\ 1 & 0 & 1 \\ 0 & 1 & 0 \end{bmatrix} \quad \begin{bmatrix} ? \end{bmatrix} \quad \begin{bmatrix} 0 \\ 1 \\ 1 \end{bmatrix}$$

Unifying Guilt-by-Association Approaches: Theorems and Fast Algorithms. Danai Koutra, et al *PKDD'11*

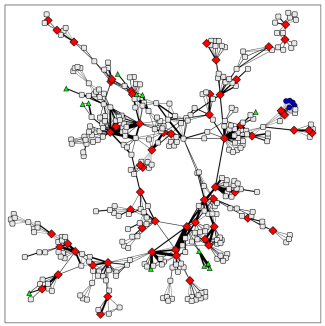
Roadmap

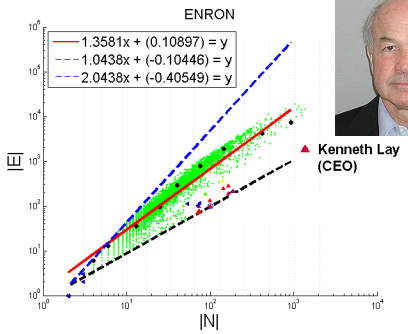
- Patterns in graphs
- Anomaly Detection
- Application: ebay fraud
- Conclusions

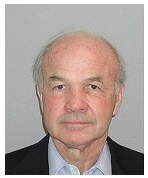
Overall Conclusions

- Roles
  - Past work in social networks ('regular', 'structural', *etc*)
  - Scalable algorithms' to find such roles
- Anomalies & patterns
  - Static (power-laws, 'six degrees')
  - Weighted (super-linearity)
  - Time-evolving (densification, -1.5 exponent)

Thank You!







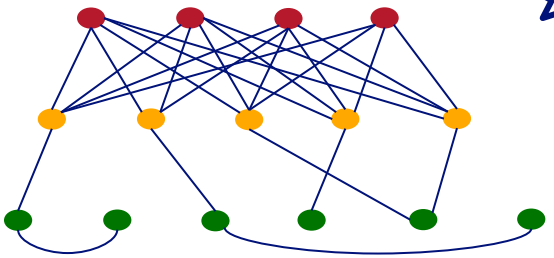
Features

Roles

Anomalies

= rare roles

Patterns



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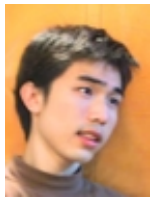
Akoglu, Leman

Chau, Polo



Kang, U

Koutra, Danai

McGlohon, Mary

Prakash, Aditya

Tong, Hanghang

Thanks to: NSF IIS-0705359, IIS-0534205, CTA-INARC; ADAMS-DARPA; Yahoo (M45), LLNL, IBM, SPRINT, Google, INTEL, HP, iLab

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